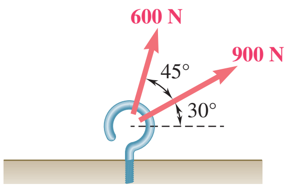
CHAPTER 2



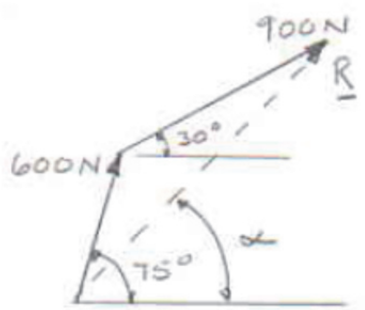
PROBLEM 2.1

Two forces are applied as shown to a hook. Determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

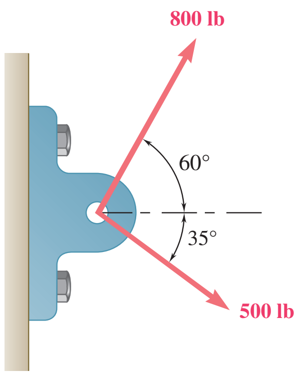
SOLUTION

(*a*) Parallelogram law:

(*b*) Triangle rule:



We measure: $R = 1391$ kN, $\alpha = 47.8°$ **R** $= 1391$ N ⦣ 47.8° ◀

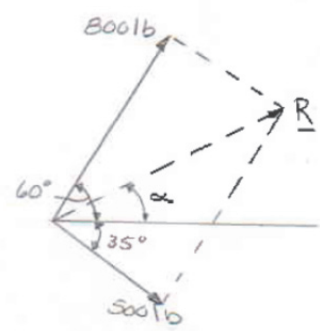


PROBLEM 2.2

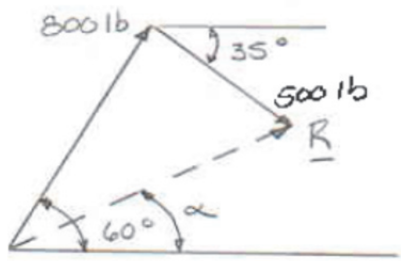
Two forces are applied as shown to a bracket support. Determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

(*a*) Parallelogram law:

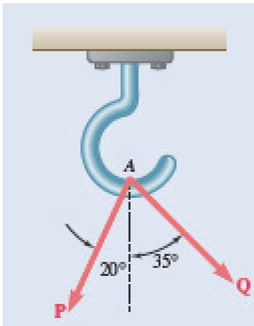


(*b*) Triangle rule:



We measure: $R = 906$ lb, $\alpha = 26.6°$ $R = 906$ lb ∠26.6° ◀

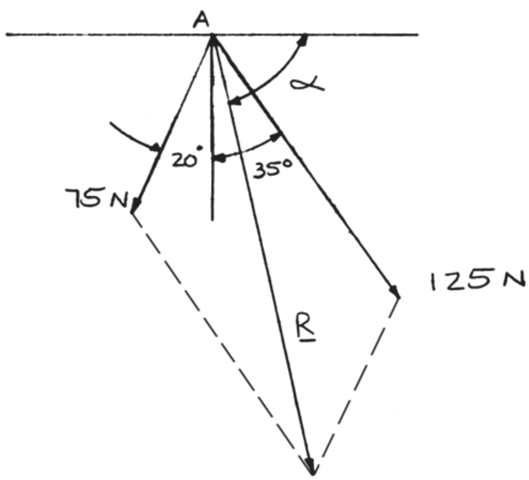
PROBLEM 2.3



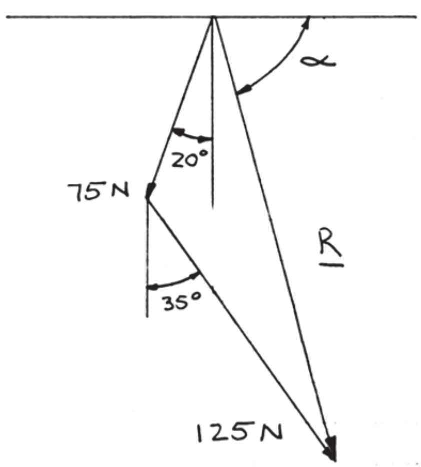
Two forces **P** and **Q** are applied as shown at Point A of a hook support. Knowing that $P = 75$ N and $Q = 125$ N, determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

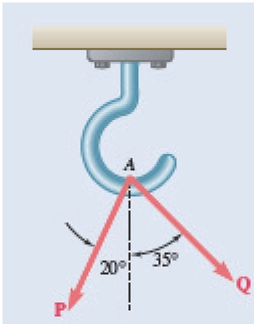
(*a*) Parallelogram law:



(*b*) Triangle rule:



We measure: $R = 179$ N, $\alpha = 75.1°$ **R** $= 179$ N ⦪ 75.1°

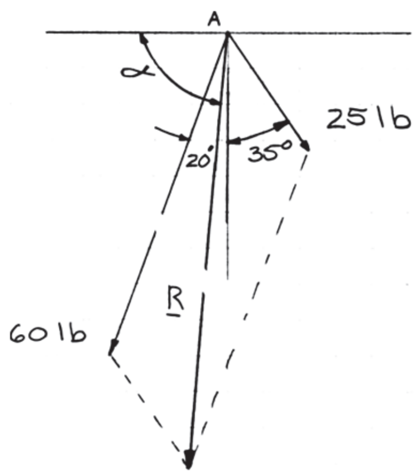


PROBLEM 2.4

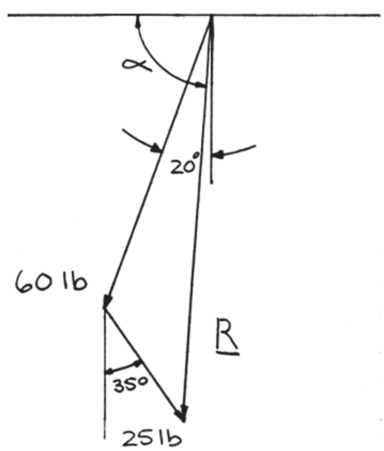
Two forces **P** and **Q** are applied as shown at Point A of a hook support. Knowing that $P = 60$ lb and $Q = 25$ lb, determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

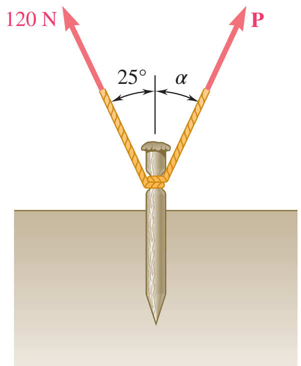
(*a*) Parallelogram law:



(*b*) Triangle rule:



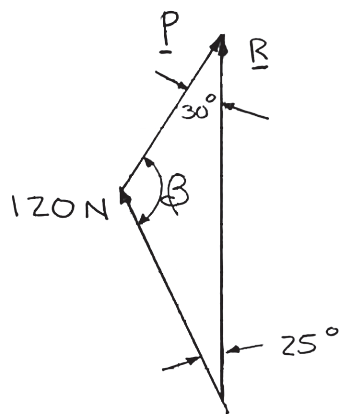
We measure: $R = 77.1$ lb, $\alpha = 85.4°$ $\mathbf{R} = 77.1$ lb ↙ 85.4°



PROBLEM 2.5

A stake is being pulled out of the ground by means of two ropes as shown. Knowing that $\alpha = 30°$, determine by trigonometry (*a*) the magnitude of the force **P** so that the resultant force exerted on the stake is vertical, (*b*) the corresponding magnitude of the resultant.

SOLUTION



Using the triangle rule and the law of sines:

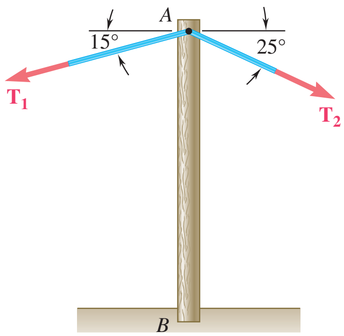
(*a*)
$$\frac{120\text{ N}}{\sin 30°} = \frac{P}{\sin 25°}$$

$P = 101.4$ N ◀

(*b*)
$$30° + \beta + 25° = 180°$$
$$\beta = 180° - 25° - 30°$$
$$= 125°$$

$$\frac{120\text{ N}}{\sin 30°} = \frac{R}{\sin 125°}$$

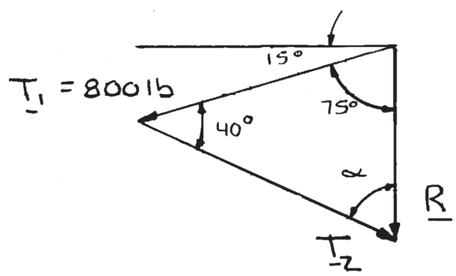
$R = 196.6$ N ◀



PROBLEM 2.6

A telephone cable is clamped at *A* to the pole *AB*. Knowing that the tension in the left-hand portion of the cable is $T_1 = 800$ lb, determine by trigonometry (*a*) the required tension T_2 in the right-hand portion if the resultant **R** of the forces exerted by the cable at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

SOLUTION



Using the triangle rule and the law of sines:

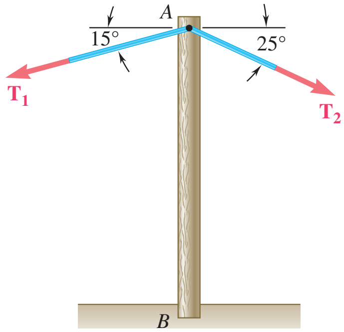
(*a*)

$$75° + 40° + \alpha = 180°$$
$$\alpha = 180° - 75° - 40°$$
$$= 65°$$

$$\frac{800 \text{ lb}}{\sin 65°} = \frac{T_2}{\sin 75°} \qquad T_2 = 853 \text{ lb} \blacktriangleleft$$

(*b*)

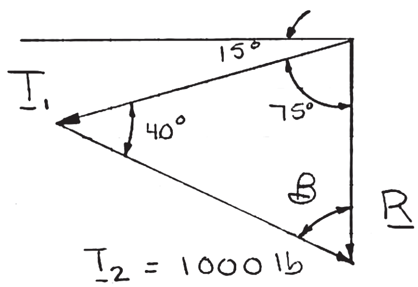
$$\frac{800 \text{ lb}}{\sin 65°} = \frac{R}{\sin 40°} \qquad R = 567 \text{ lb} \blacktriangleleft$$



PROBLEM 2.7

A telephone cable is clamped at A to the pole AB. Knowing that the tension in the right-hand portion of the cable is $T_2 = 1000$ lb, determine by trigonometry (a) the required tension T_1 in the left-hand portion if the resultant **R** of the forces exerted by the cable at A is to be vertical, (b) the corresponding magnitude of **R**.

SOLUTION



Using the triangle rule and the law of sines:

(a)

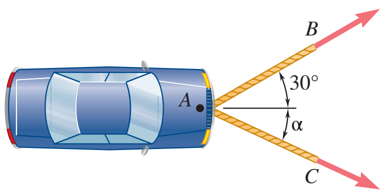
$$75° + 40° + \beta = 180°$$
$$\beta = 180° - 75° - 40°$$
$$= 65°$$

$$\frac{1000 \text{ lb}}{\sin 75°} = \frac{T_1}{\sin 65°} \qquad T_1 = 938 \text{ lb} ◀$$

(b)

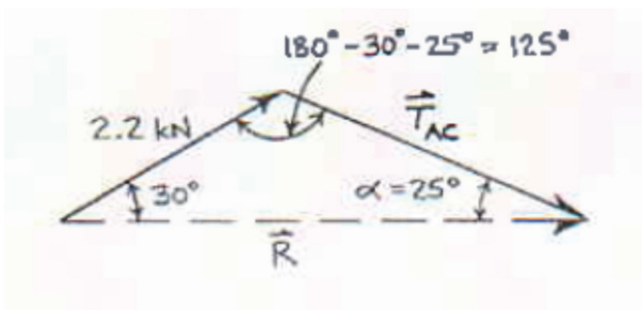
$$\frac{1000 \text{ lb}}{\sin 75°} = \frac{R}{\sin 40°} \qquad R = 665 \text{ lb} ◀$$

PROBLEM 2.8



A disabled automobile is pulled by means of two ropes as shown. The tension in rope *AB* is 2.2 kN, and the angle α is 25°. Knowing that the resultant of the two forces applied at *A* is directed along the axis of the automobile, determine by trigonometry (*a*) the tension in rope *AC*, (*b*) the magnitude of the resultant of the two forces applied at *A*.

SOLUTION



Using the law of sines:

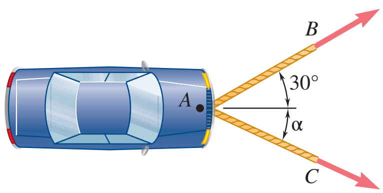
$$\frac{T_{AC}}{\sin 30°} = \frac{R}{\sin 125°} = \frac{2.2 \text{ kN}}{\sin 25°}$$

$$T_{AC} = 2.603 \text{ kN}$$
$$R = 4.264 \text{ kN}$$

(*a*) $T_{AC} = 2.60 \text{ kN}$ ◀

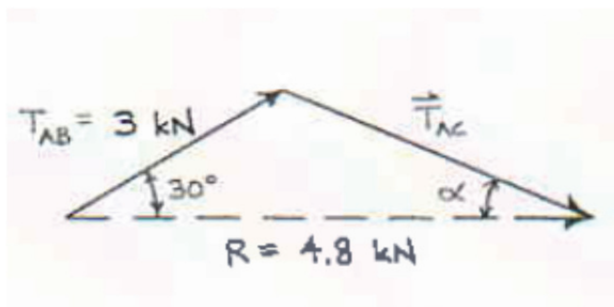
(*b*) $R = 4.26 \text{ kN}$ ◀

PROBLEM 2.9



A disabled automobile is pulled by means of two ropes as shown. Knowing that the tension in rope AB is 3 kN, determine by trigonometry the tension in rope AC and the value of α so that the resultant force exerted at A is a 4.8-kN force directed along the axis of the automobile.

SOLUTION



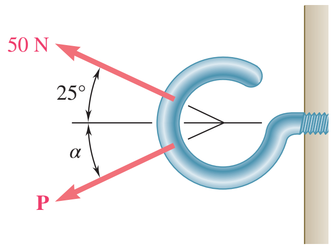
Using the law of cosines: $T_{AC}^2 = (3\text{ kN})^2 + (4.8\text{ kN})^2 - 2(3\text{ kN})(4.8\text{ kN})\cos 30°$

$$T_{AC} = 2.6643\text{ kN}$$

Using the law of sines: $\dfrac{\sin\alpha}{3\text{ kN}} = \dfrac{\sin 30°}{2.6643\text{ kN}}$

$$\alpha = 34.3°$$

$\mathbf{T}_{AC} = 2.66\text{ kN} \searrow 34.3°$ ◀

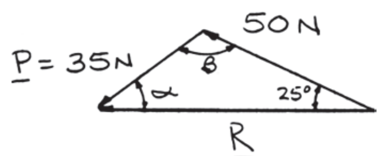


PROBLEM 2.10

Two forces are applied as shown to a hook support. Knowing that the magnitude of **P** is 35 N, determine by trigonometry (*a*) the required angle α if the resultant **R** of the two forces applied to the support is to be horizontal, (*b*) the corresponding magnitude of **R**.

SOLUTION

Using the triangle rule and law of sines:



(*a*)

$$\frac{\sin\alpha}{50\text{ N}} = \frac{\sin 25^\circ}{35\text{ N}}$$

$$\sin\alpha = 0.60374$$

$$\alpha = 37.138^\circ$$

$\alpha = 37.1^\circ$ ◀

(*b*)

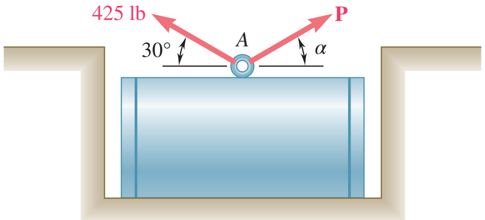
$$\alpha + \beta + 25^\circ = 180^\circ$$

$$\beta = 180^\circ - 25^\circ - 37.138^\circ$$

$$= 117.862^\circ$$

$$\frac{R}{\sin 117.862^\circ} = \frac{35\text{ N}}{\sin 25^\circ}$$

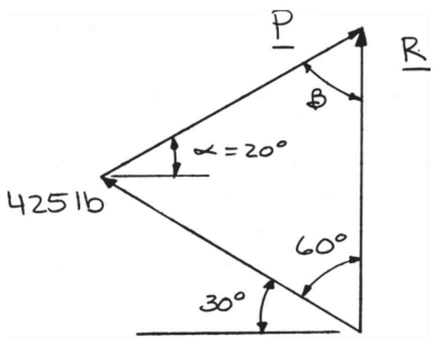
$R = 73.2$ N ◀



PROBLEM 2.11

A steel tank is to be positioned in an excavation. Knowing that $\alpha = 20°$, determine by trigonometry (*a*) the required magnitude of the force **P** if the resultant **R** of the two forces applied at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

SOLUTION



Using the triangle rule and the law of sines:

(*a*) $$\beta + 50° + 60° = 180°$$

$$\beta = 180° - 50° - 60°$$

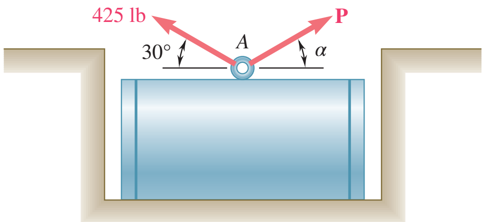
$$= 70°$$

$$\frac{425 \text{ lb}}{\sin 70°} = \frac{P}{\sin 60°}$$

$P = 392$ lb ◀

(*b*) $$\frac{425 \text{ lb}}{\sin 70°} = \frac{R}{\sin 50°}$$

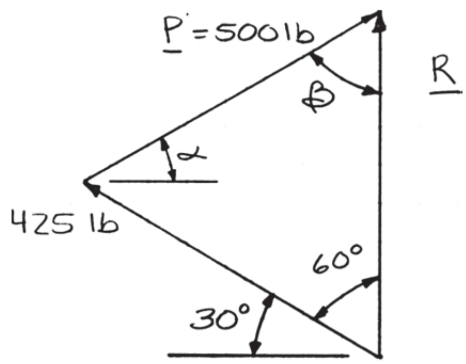
$R = 346$ lb ◀



PROBLEM 2.12

A steel tank is to be positioned in an excavation. Knowing that the magnitude of **P** is 500 lb, determine by trigonometry (*a*) the required angle α if the resultant **R** of the two forces applied at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

SOLUTION



Using the triangle rule and the law of sines:

(*a*)

$$(\alpha + 30°) + 60° + \beta = 180°$$

$$\beta = 180° - (\alpha + 30°) - 60°$$

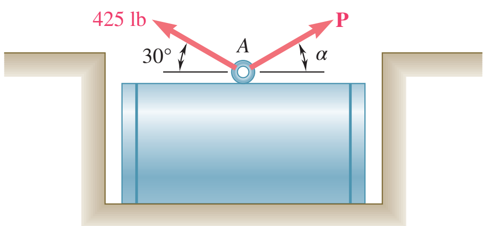
$$\beta = 90° - \alpha$$

$$\frac{\sin(90° - \alpha)}{425 \text{ lb}} = \frac{\sin 60°}{500 \text{ lb}}$$

$$90° - \alpha = 47.402° \qquad \alpha = 42.6° \blacktriangleleft$$

(*b*)

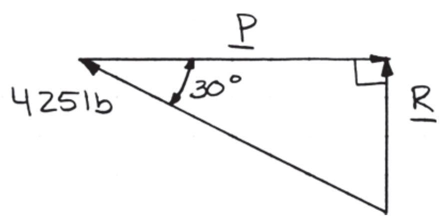
$$\frac{R}{\sin(42.598° + 30°)} = \frac{500 \text{ lb}}{\sin 60°} \qquad R = 551 \text{ lb} \blacktriangleleft$$



PROBLEM 2.13

A steel tank is to be positioned in an excavation. Determine by trigonometry (*a*) the magnitude and direction of the smallest force **P** for which the resultant **R** of the two forces applied at *A* is vertical, (*b*) the corresponding magnitude of **R**.

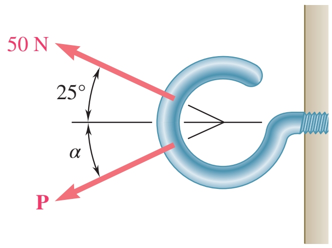
SOLUTION



The smallest force *P* will be perpendicular to *R*.

(*a*) $P = (425 \text{ lb})\cos 30°$ $\qquad$ $\mathbf{P} = 368 \text{ lb} \longrightarrow$ ◀

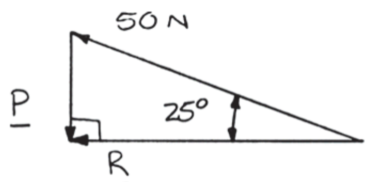
(*b*) $R = (425 \text{ lb})\sin 30°$ $\qquad$ $R = 213 \text{ lb}$ ◀



PROBLEM 2.14

For the hook support of Prob. 2.10, determine by trigonometry (*a*) the magnitude and direction of the smallest force **P** for which the resultant **R** of the two forces applied to the support is horizontal, (*b*) the corresponding magnitude of **R**.

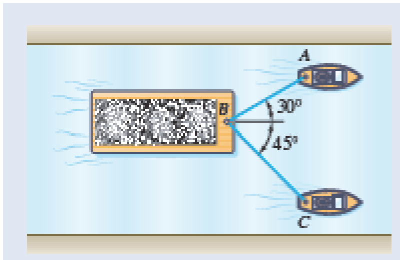
SOLUTION



The smallest force P will be perpendicular to R.

(*a*) $P = (50\text{ N})\sin 25°$ $\mathbf{P} = 21.1\text{ N}\downarrow$ ◀

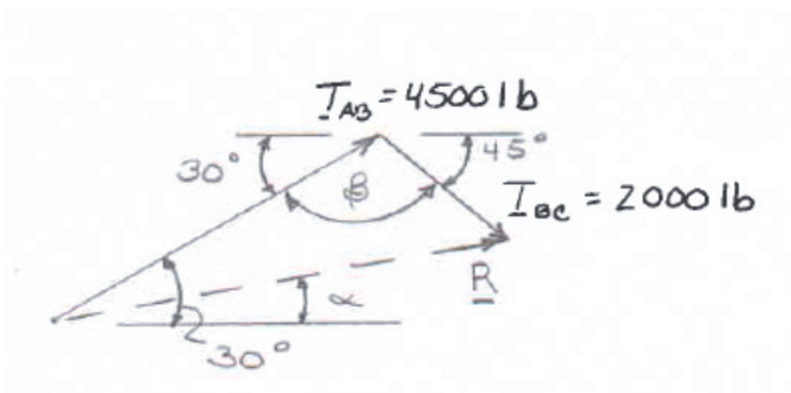
(*b*) $R = (50\text{ N})\cos 25°$ $R = 45.3\text{ N}$ ◀



PROBLEM 2.15

The barge *B* is pulled by two tugboats *A* and *C*. At a given instant the tension in cable *AB* is 4500 lb and the tension in cable *BC* is 2000 lb. Determine by trigonometry the magnitude and direction of the resultant of the two forces applied at *B* at that instant.

SOLUTION



Using the law of cosines:

$$\beta = 180° - 30° - 45°$$
$$\beta = 105°$$

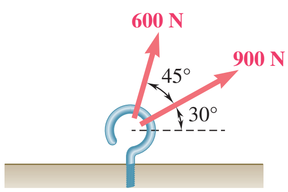
$$R^2 = (4500 \text{ lb})^2 + (2000 \text{ lb})^2 - 2(4500 \text{ lb})(2000 \text{ lb})\cos 105°$$
$$R = 5380 \text{ lb}$$

Using the law of sines:

$$\frac{R}{\sin\beta} = \frac{2000 \text{ lb}}{\sin(30° - \alpha)}$$
$$\frac{5380 \text{ lb}}{\sin 105°} = \frac{2000 \text{ lb}}{\sin(30° - \alpha)}$$

$$\alpha = 8.94°$$

R = 5380 lb ⦣ 8.94° ◀



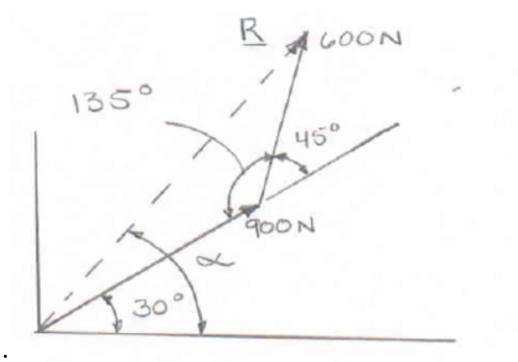
PROBLEM 2.16

Solve Prob. 2.1 by trigonometry.

PROBLEM 2.1

Two forces are applied as shown to a hook. Determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION



Using the law of cosines:

$$R^2 = (900\,\text{N})^2 + (600\,\text{N})^2 - 2(900\,\text{N})(600\,\text{N})\cos(135°)$$

$$R = 1390.57\text{N}$$

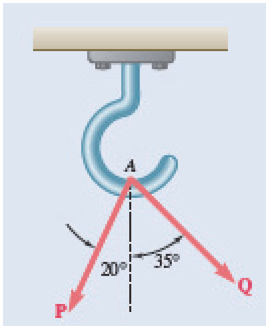
Using the law of sines:

$$\frac{\sin(\alpha - 30°)}{600\text{N}} = \frac{\sin(135°)}{1390.57\text{N}}$$

$$\alpha - 30° = 17.7642°$$

$$\alpha = 47.764°$$

$\mathbf{R} = 1391\text{N} \measuredangle 47.8°$ ◀



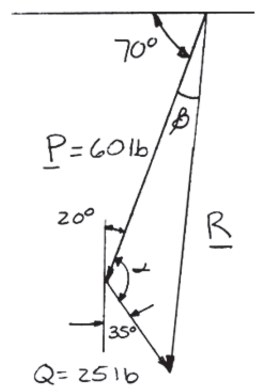
PROBLEM 2.17

Solve Problem 2.4 by trigonometry.

PROBLEM 2.4 Two forces **P** and **Q** are applied as shown at Point *A* of a hook support. Knowing that $P = 60$ lb and $Q = 25$ lb, determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

Using the triangle rule and the law of cosines:



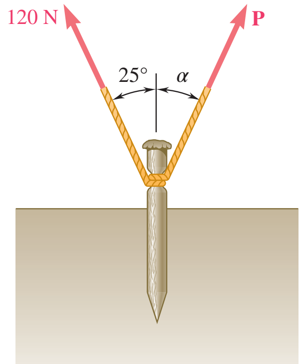
$$20° + 35° + \alpha = 180°$$
$$\alpha = 125°$$
$$R^2 = P^2 + Q^2 - 2PQ\cos\alpha$$
$$R^2 = (60\text{ lb})^2 + (25\text{ lb})^2 - 2(60\text{ lb})(25\text{ lb})\cos 125°$$
$$R^2 = 3600 + 625 + 3000(0.5736)$$
$$R = 77.108\text{ lb}$$

Using the law of sines:

$$\frac{\sin\beta}{25\text{ lb}} = \frac{\sin 125°}{77.108\text{ lb}}$$
$$\beta = 15.402°$$

$$70° + \beta = 85.402°$$

$\mathbf{R} = 77.1$ lb ↗ 85.4° ◀

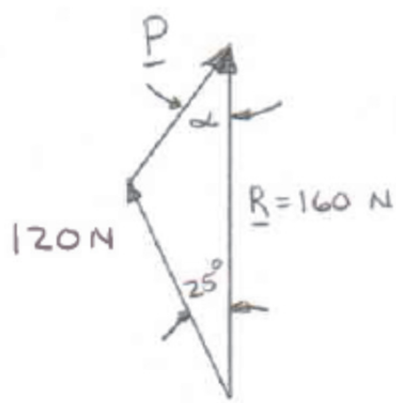


PROBLEM 2.18

For the stake of Prob. 2.5, knowing that the tension in one rope is 120 N, determine by trigonometry the magnitude and direction of the force **P** so that the resultant is a vertical force of 160 N.

PROBLEM 2.5 A stake is being pulled out of the ground by means of two ropes as shown. Knowing that $\alpha = 30°$, determine by trigonometry (*a*) the magnitude of the force **P** so that the resultant force exerted on the stake is vertical, (*b*) the corresponding magnitude of the resultant.

SOLUTION



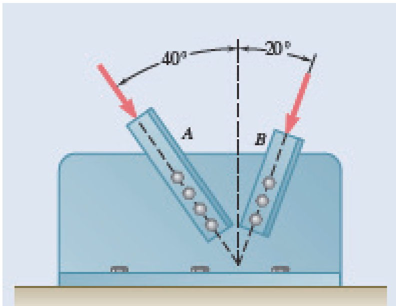
Using the laws of cosines and sines:

$$P^2 = (120\text{ N})^2 + (160\text{ N})^2 - 2(120\text{ N})(160\text{ N})\cos 25°$$
$$P = 72.096\text{ N}$$

And

$$\frac{\sin\alpha}{120\text{ N}} = \frac{\sin 25°}{72.096\text{ N}}$$
$$\sin\alpha = 0.70343$$
$$\alpha = 44.703°$$

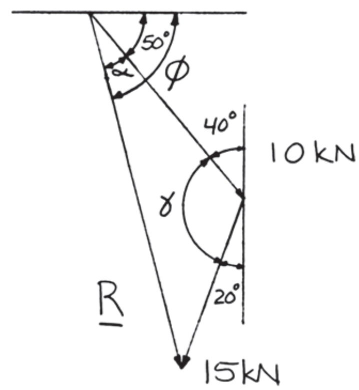
$\mathbf{P} = 72.1\text{ N}$ ⦣ 44.7° ◀



PROBLEM 2.19

Two structural members *A* and *B* are bolted to a bracket as shown. Knowing that both members are in compression and that the force is 10 kN in member *A* and 15 kN in member *B*, determine by trigonometry the magnitude and direction of the resultant of the forces applied to the bracket by members *A* and *B*.

SOLUTION



Using the force triangle and the laws of cosines and sines

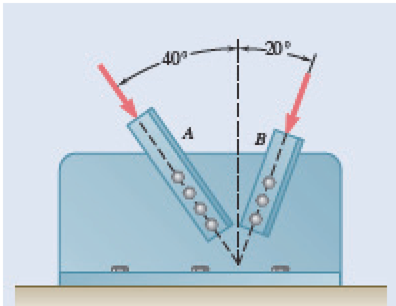
We have
$$\gamma = 180° - (40° + 20°) = 120°$$

Then
$$R^2 = (10 \text{ kN})^2 + (15 \text{ kN})^2 - 2(10 \text{ kN})(15 \text{ kN})\cos 120° = 475 \text{ kN}^2$$
$$R = 21.794 \text{ kN}$$

and
$$\frac{15 \text{ kN}}{\sin\alpha} = \frac{21.794 \text{ kN}}{\sin 120°}$$
$$\sin\alpha = \left(\frac{15 \text{ kN}}{21.794 \text{ kN}}\right)\sin 120° = 0.59605$$
$$\alpha = 36.588°$$

Hence:
$$\phi = \alpha + 50° = 86.588°$$

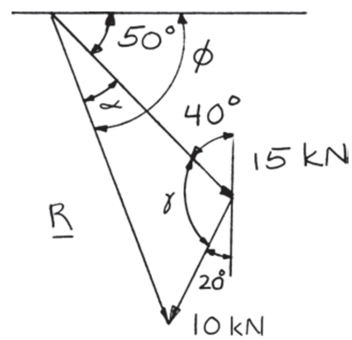
R = 21.8 kN ⦪ 86.6°



PROBLEM 2.20

Two structural members *A* and *B* are bolted to a bracket as shown. Knowing that both members are in compression and that the force is 15 kN in member *A* and 10 kN in member *B*, determine by trigonometry the magnitude and direction of the resultant of the forces applied to the bracket by members *A* and *B*.

SOLUTION



Using the force triangle and the laws of cosines and sines:

We have $\gamma = 180° - (40° + 20°)$

$= 120°$

Then

$$R^2 = (15 \text{ kN})^2 + (10 \text{ kN})^2 - 2(15 \text{ kN})(10 \text{ kN})\cos 120°$$

$$= 475 \text{ kN}^2$$

$$R = 21.794 \text{ kN}$$

and

$$\frac{10 \text{ kN}}{\sin\alpha} = \frac{21.794 \text{ kN}}{\sin 120°}$$

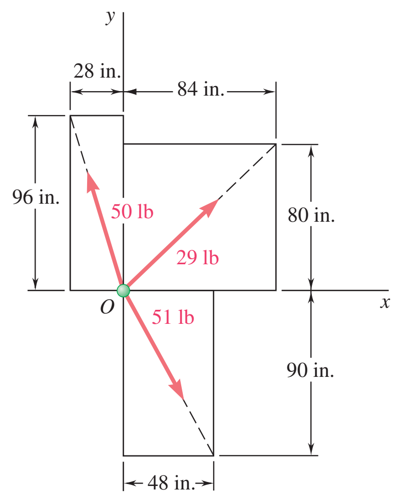
$$\sin\alpha = \left(\frac{10 \text{ kN}}{21.794 \text{ kN}}\right)\sin 120°$$

$$= 0.39737$$

$$\alpha = 23.414$$

Hence: $\phi = \alpha + 50° = 73.414$ **R** $= 21.8$ kN ⦪ $73.4°$

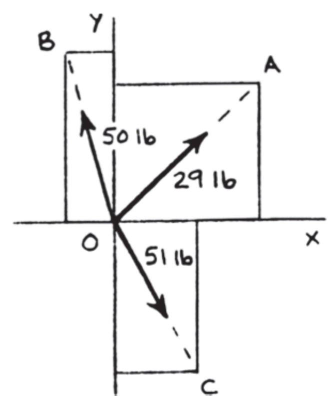
PROBLEM 2.21



Determine the x and y components of each of the forces shown.

SOLUTION

Compute the following distances:



$$OA = \sqrt{(84)^2 + (80)^2}$$
$$= 116 \text{ in.}$$
$$OB = \sqrt{(28)^2 + (96)^2}$$
$$= 100 \text{ in.}$$
$$OC = \sqrt{(48)^2 + (90)^2}$$
$$= 102 \text{ in.}$$

29-lb Force:

$$F_x = +(29 \text{ lb})\frac{84}{116} \qquad F_x = +21.0 \text{ lb} ◀$$

$$F_y = +(29 \text{ lb})\frac{80}{116} \qquad F_y = +20.0 \text{ lb} ◀$$

50-lb Force:

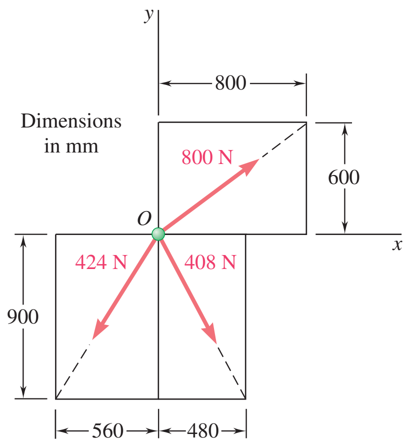
$$F_x = -(50 \text{ lb})\frac{28}{100} \qquad F_x = -14.00 \text{ lb} ◀$$

$$F_y = +(50 \text{ lb})\frac{96}{100} \qquad F_y = +48.0 \text{ lb} ◀$$

51-lb Force:

$$F_x = +(51 \text{ lb})\frac{48}{102} \qquad F_x = +24.0 \text{ lb} ◀$$

$$F_y = -(51 \text{ lb})\frac{90}{102} \qquad F_y = -45.0 \text{ lb} ◀$$

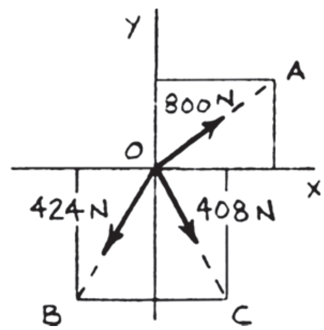


PROBLEM 2.22

Determine the x and y components of each of the forces shown.

SOLUTION

Compute the following distances:



$$OA = \sqrt{(600)^2 + (800)^2} = 1000 \text{ mm}$$

$$OB = \sqrt{(560)^2 + (900)^2} = 1060 \text{ mm}$$

$$OC = \sqrt{(480)^2 + (900)^2} = 1020 \text{ mm}$$

800-N Force:

$$F_x = +(800 \text{ N})\frac{800}{1000} \qquad F_x = +640 \text{ N} \blacktriangleleft$$

$$F_y = +(800 \text{ N})\frac{600}{1000} \qquad F_y = +480 \text{ N} \blacktriangleleft$$

424-N Force:

$$F_x = -(424 \text{ N})\frac{560}{1060} \qquad F_x = -224 \text{ N} \blacktriangleleft$$

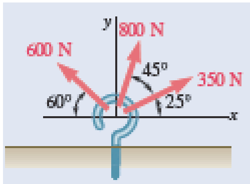
$$F_y = -(424 \text{ N})\frac{900}{1060} \qquad F_y = -360 \text{ N} \blacktriangleleft$$

408-N Force:

$$F_x = +(408 \text{ N})\frac{480}{1020} \qquad F_x = +192.0 \text{ N} \blacktriangleleft$$

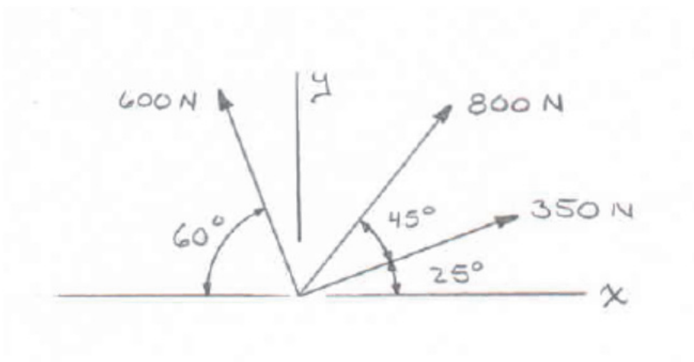
$$F_y = -(408 \text{ N})\frac{900}{1020} \qquad F_y = -360 \text{ N} \blacktriangleleft$$

PROBLEM 2.23



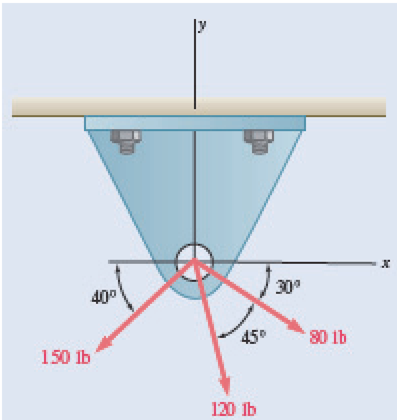
Determine the x and y components of each of the forces shown.

SOLUTION



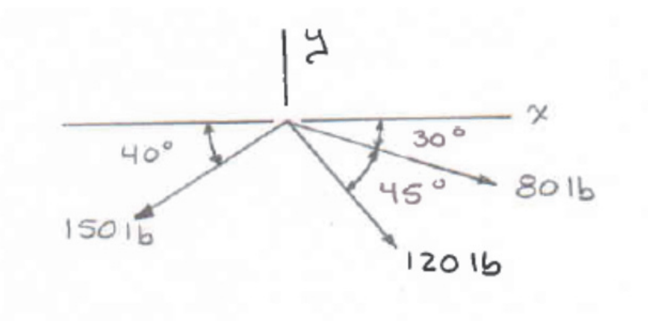
350-N Force:	$F_x = +(350\text{ N})\cos 25°$	$F_x = +317\text{ N}$ ◀
	$F_y = +(350\text{ N})\sin 25°$	$F_y = +147.9\text{ N}$ ◀
800-N Force:	$F_x = +(800\text{ N})\cos 70°$	$F_x = +274\text{ N}$ ◀
	$F_y = +(800\text{ N})\sin 70°$	$F_y = +752\text{ N}$ ◀
600-N Force:	$F_x = -(600\text{ N})\cos 60°$	$F_x = -300\text{ N}$ ◀
	$F_y = +(600\text{ N})\sin 60°$	$F_y = +520\text{ N}$ ◀

PROBLEM 2.24



Determine the x and y components of each of the forces shown.

SOLUTION



80-lb Force: $F_x = +(80\text{ lb})\cos 30°$ $\qquad F_x = +69.3\text{ lb}$ ◀

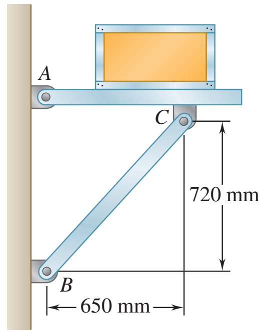
$F_y = -(80\text{ lb})\sin 30°$ $\qquad F_y = -40.0\text{ lb}$ ◀

120-lb Force: $F_x = +(120\text{ lb})\cos 75°$ $\qquad F_x = +31.1\text{ lb}$ ◀

$F_y = -(120\text{ lb})\sin 75°$ $\qquad F_y = -115.9\text{ lb}$ ◀

150-lb Force: $F_x = -(150\text{ lb})\cos 40°$ $\qquad F_x = -114.9\text{ lb}$ ◀

$F_y = -(150\text{ lb})\sin 40°$ $\qquad F_y = -96.4\text{ lb}$ ◀

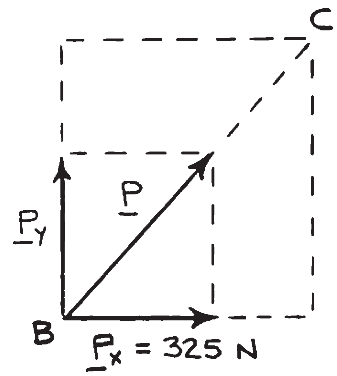


PROBLEM 2.25

Member *BC* exerts on member *AC* a force **P** directed along line *BC*. Knowing that **P** must have a 325-N horizontal component, determine (*a*) the magnitude of the force **P**, (*b*) its vertical component.

SOLUTION

$$BC = \sqrt{(650\text{ mm})^2 + (720\text{ mm})^2}$$
$$= 970\text{ mm}$$



(*a*)

$$P_x = P\left(\frac{650}{970}\right)$$

or

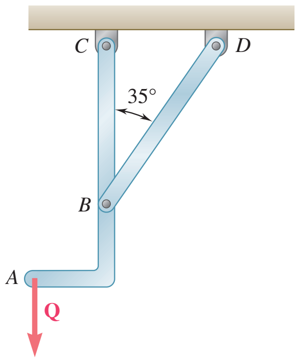
$$P = P_x\left(\frac{970}{650}\right)$$
$$= 325\text{ N}\left(\frac{970}{650}\right)$$
$$= 485\text{ N}$$

$P = 485$ N ◀

(*b*)

$$P_y = P\left(\frac{720}{970}\right)$$
$$= 485\text{ N}\left(\frac{720}{970}\right)$$
$$= 360\text{ N}$$

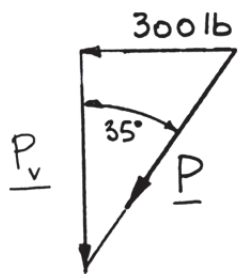
$P_y = 970$ N ◀



PROBLEM 2.26

Member *BD* exerts on member *ABC* a force **P** directed along line *BD*. Knowing that **P** must have a 300-lb horizontal component, determine (*a*) the magnitude of the force **P**, (*b*) its vertical component.

SOLUTION

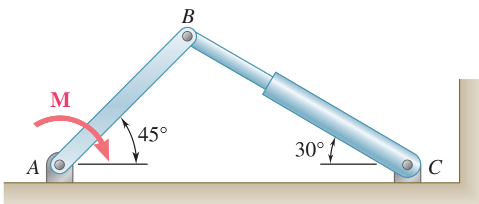


(*a*) $$P \sin 35° = 300 \text{ lb}$$

$$P = \frac{300 \text{ lb}}{\sin 35°}$$ $P = 523$ lb ◀

(*b*) Vertical component $$P_v = P \cos 35°$$

$$= (523 \text{ lb}) \cos 35°$$ $P_v = 428$ lb ◀

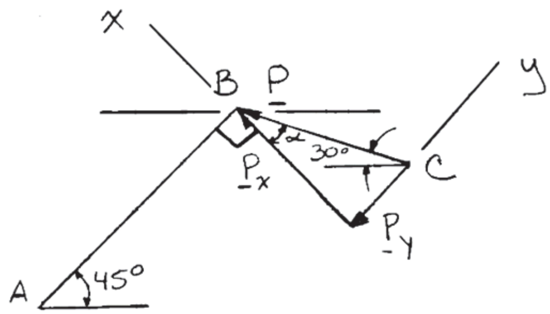


PROBLEM 2.27

The hydraulic cylinder BC exerts on member AB a force $\mathbf{P}$ directed along line BC. Knowing that $\mathbf{P}$ must have a 600-N component perpendicular to member AB, determine (*a*) the magnitude of the force $\mathbf{P}$, (*b*) its component along line AB.

SOLUTION

$$180° = 45° + \alpha + 90° + 30°$$
$$\alpha = 180° - 45° - 90° - 30°$$
$$= 15°$$



(*a*)

$$\cos\alpha = \frac{P_x}{P}$$
$$P = \frac{P_x}{\cos\alpha}$$
$$= \frac{600 \text{ N}}{\cos 15°}$$
$$= 621.17 \text{ N}$$

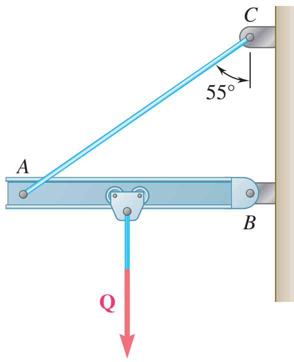
$P = 621$ N ◀

(*b*)

$$\tan\alpha = \frac{P_y}{P_x}$$
$$P_y = P_x \tan\alpha$$
$$= (600 \text{ N})\tan 15°$$
$$= 160.770 \text{ N}$$

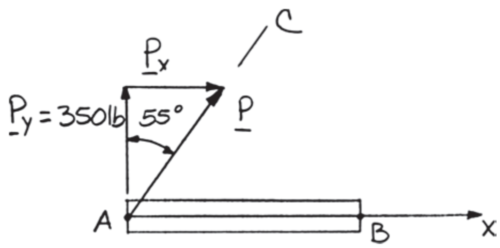
$P_y = 160.8$ N ◀

PROBLEM 2.28



Cable AC exerts on beam AB a force **P** directed along line AC. Knowing that **P** must have a 350-lb vertical component, determine (*a*) the magnitude of the force **P**, (*b*) its horizontal component.

SOLUTION



(*a*)

$$P = \frac{P_y}{\cos 55°}$$

$$= \frac{350 \text{ lb}}{\cos 55°}$$

$$= 610.21 \text{ lb}$$

$P = 610 \text{ lb}$ ◀

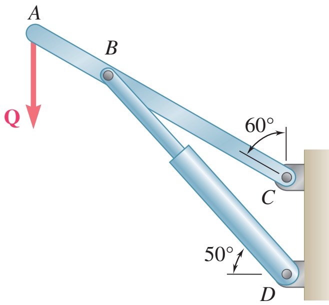
(*b*)

$$P_x = P \sin 55°$$

$$= (610.21 \text{ lb})\sin 55°$$

$$= 499.85 \text{ lb}$$

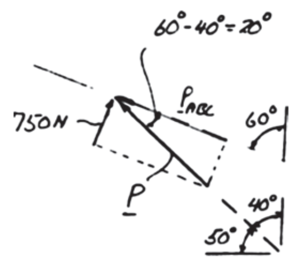
$P_x = 500 \text{ lb}$ ◀



PROBLEM 2.29

The hydraulic cylinder BD exerts on member ABC a force **P** directed along line BD. Knowing that **P** must have a 750-N component perpendicular to member ABC, determine (*a*) the magnitude of the force **P**, (*b*) its component parallel to ABC.

SOLUTION



(*a*)

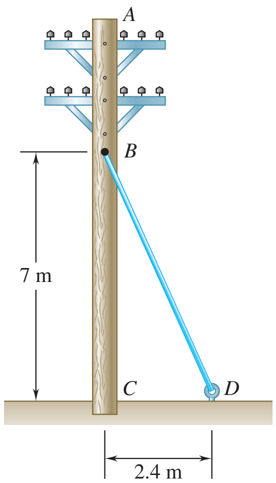
$$750\text{ N} = P\sin 20°$$

$$P = 2192.9\text{ N} \qquad P = 2190\text{ N} ◀$$

(*b*)

$$P_{ABC} = P\cos 20°$$

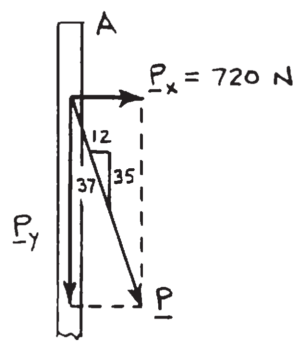
$$= (2192.9\text{ N})\cos 20° \qquad P_{ABC} = 2060\text{ N} ◀$$



PROBLEM 2.30

The guy wire *BD* exerts on the telephone pole *AC* a force **P** directed along *BD*. Knowing that **P** must have a 720-N component perpendicular to the pole *AC*, determine (*a*) the magnitude of the force **P**, (*b*) its component along line *AC*.

SOLUTION



(*a*)

$$P = \frac{37}{12}P_x$$
$$= \frac{37}{12}(720\text{ N})$$
$$= 2220\text{ N}$$

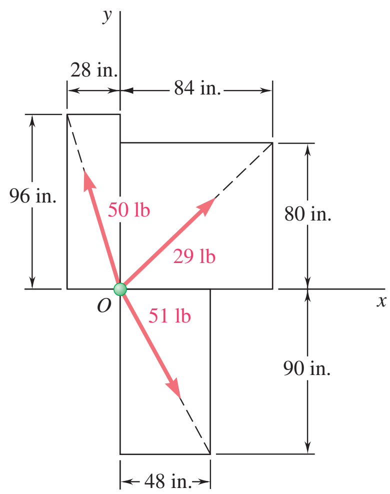
$P = 2.22$ kN ◀

(*b*)

$$P_y = \frac{35}{12}P_x$$
$$= \frac{35}{12}(720\text{ N})$$
$$= 2100\text{ N}$$

$P_y = 2.10$ kN ◀

PROBLEM 2.31



Determine the resultant of the three forces of Problem 2.21.

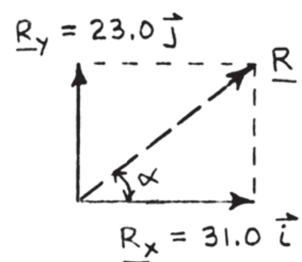
PROBLEM 2.21 Determine the x and y components of each of the forces shown.

SOLUTION

Components of the forces were determined in Problem 2.21:

Force	x Comp. (lb)	y Comp. (lb)
29 lb	+21.0	+20.0
50 lb	−14.00	+48.0
51 lb	+24.0	−45.0
	$R_x = +31.0$	$R_y = +23.0$

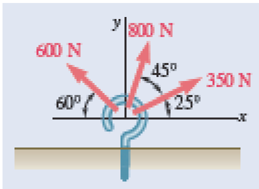
$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$
$$= (31.0 \text{ lb})\mathbf{i} + (23.0 \text{ lb})\mathbf{j}$$



$$\tan\alpha = \frac{R_y}{R_x}$$
$$= \frac{23.0}{31.0}$$
$$\alpha = 36.573°$$
$$R = \frac{23.0 \text{ lb}}{\sin(36.573°)}$$
$$= 38.601 \text{ lb}$$

R = 38.6 lb ∠36.6° ◀

PROBLEM 2.32



Determine the resultant of the three forces of Problem 2.23.

PROBLEM 2.23 Determine the x and y components of each of the forces shown.

SOLUTION

Components of the forces were determined in Problem 2.23:

Force	x Comp. (N)	y Comp. (N)
350 N	+317	+147.9
800 N	+274	+752
600 N	−300	+520
	$R_x = +291$	$R_y = +1419.9$

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$
$$= (291\text{ N})\mathbf{i} + (1419.9\text{ N})\mathbf{j}$$

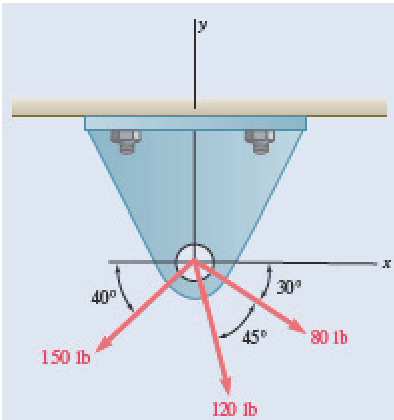
$$\tan\alpha = \frac{R_y}{R_x}$$
$$= \frac{1419.9\text{ N}}{291\text{ N}}$$
$$\alpha = 78.418^\circ$$

$$R = \frac{1419.9\text{ N}}{\sin(78.418^\circ)}$$

$$= 1449\text{ N}$$

$\mathbf{R} = 1449\text{ N} \measuredangle 78.4^\circ$ ◀

PROBLEM 2.33



Determine the resultant of the three forces of Problem 2.24.

PROBLEM 2.24 Determine the x and y components of each of the forces shown.

SOLUTION

Components of the forces were determined in Problem 2.24:

Force	x Comp. (lb)	y Comp. (lb)
80 lb	+69.3	−40.0
120 lb	+31.1	−115.9
150 lb	−114.9	−96.4
	$R_x = -14.50$	$R_y = -252.3$

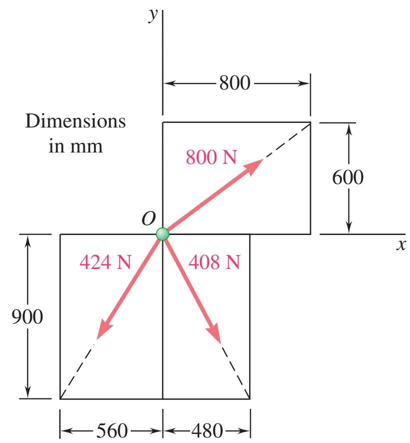
$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$
$$= -(14.50 \text{ lb})\mathbf{i} - (252.3 \text{ lb})\mathbf{j}$$

$$\tan\beta = \frac{R_x}{R_y}$$
$$= \frac{-14.50}{-252.3}$$
$$\beta = 3.289°$$

$$R = \frac{252.3 \text{ lb}}{\cos(3.289°)}$$

$$= 253 \text{ lb}$$

R = 253 lb ⦨ 86.7° ◀



PROBLEM 2.34

Determine the resultant of the three forces of Problem 2.22.

PROBLEM 2.22 Determine the x and y components of each of the forces shown.

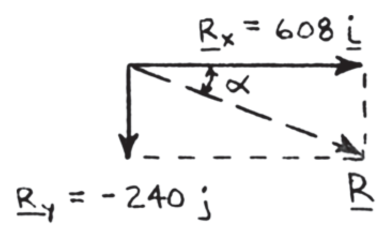
SOLUTION

Components of the forces were determined in Problem 2.22:

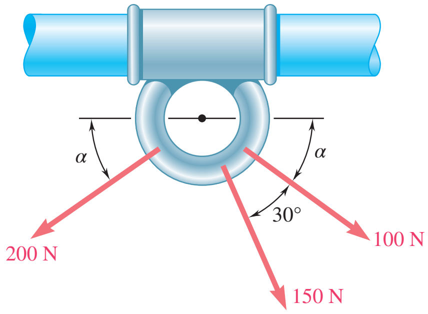
Force	x Comp. (N)	y Comp. (N)
800 lb	+640	+480
424 lb	−224	−360
408 lb	+192	−360
	$R_x = +608$	$R_y = -240$

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$
$$= (608 \text{ lb})\mathbf{i} + (-240 \text{ lb})\mathbf{j}$$

$$\tan\alpha = \frac{R_y}{R_x}$$
$$= \frac{240}{608}$$
$$\alpha = 21.541°$$
$$R = \frac{240 \text{ N}}{\sin(21.541°)}$$
$$= 653.65 \text{ N}$$



$\mathbf{R} = 654$ N ⦫ 21.5° ◀



PROBLEM 2.35

Knowing that $\alpha = 35°$, determine the resultant of the three forces shown.

SOLUTION

100-N Force:

$$F_x = +(100\text{ N})\cos 35° = +81.915\text{ N}$$
$$F_y = -(100\text{ N})\sin 35° = -57.358\text{ N}$$

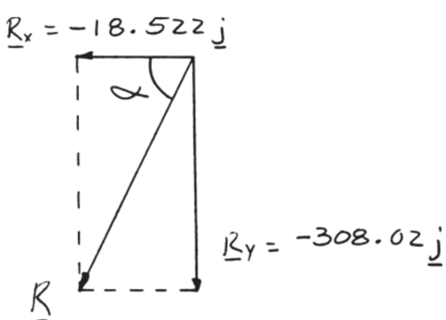
150-N Force:

$$F_x = +(150\text{ N})\cos 65° = +63.393\text{ N}$$
$$F_y = -(150\text{ N})\sin 65° = -135.946\text{ N}$$

200-N Force:

$$F_x = -(200\text{ N})\cos 35° = -163.830\text{ N}$$
$$F_y = -(200\text{ N})\sin 35° = -114.715\text{ N}$$

Force	x Comp. (N)	y Comp. (N)
100 N	+81.915	−57.358
150 N	+63.393	−135.946
200 N	−163.830	−114.715
	$R_x = -18.522$	$R_y = -308.02$

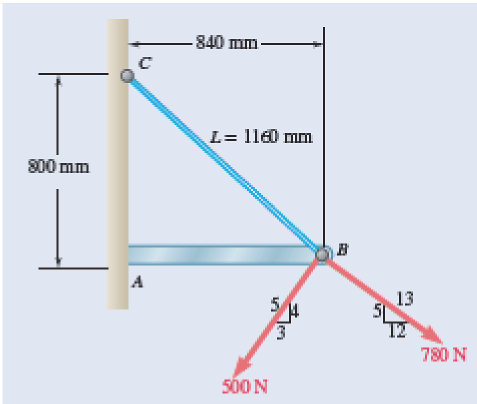


$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$
$$= (-18.522\text{ N})\mathbf{i} + (-308.02\text{ N})\mathbf{j}$$

$$\tan\alpha = \frac{R_y}{R_x}$$
$$= \frac{308.02}{18.522}$$
$$\alpha = 86.559°$$

$$R = \frac{308.02\text{ N}}{\sin 86.559}$$

R = 309 N ↙ 86.6° ◀



PROBLEM 2.36

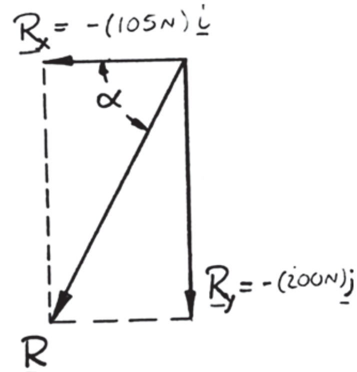
Knowing that the tension in cable *BC* is 725 N, determine the resultant of the three forces exerted at Point *B* of beam *AB*.

SOLUTION

Cable *BC* Force:

$$F_x = -(725\text{ N})\frac{840}{1160} = -525\text{ N}$$

$$F_y = (725\text{ N})\frac{840}{1160} = 500\text{ N}$$



500-N Force:

$$F_x = -(500\text{ N})\frac{3}{5} = -300\text{ N}$$

$$F_y = -(500\text{ N})\frac{4}{5} = -400\text{ N}$$

780-N Force:

$$F_x = (780\text{ N})\frac{12}{13} = 720\text{ N}$$

$$F_y = -(780\text{ N})\frac{5}{13} = -300\text{ N}$$

and

$$R_x = \Sigma F_x = -105\text{ N}$$

$$R_y = \Sigma F_y = -200\text{ N}$$

$$R = \sqrt{(-105\text{ N})^2 + (-200\text{ N})^2}$$

$$= 225.89\text{ N}$$

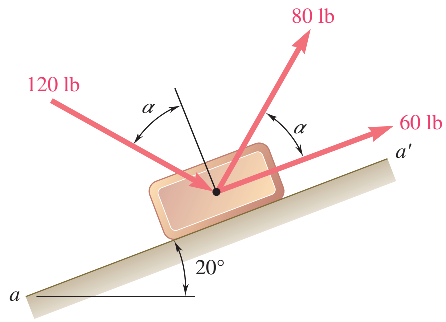
Further:

$$\tan\alpha = \frac{200}{105}$$

$$\alpha = \tan^{-1}\frac{200}{105}$$

$$= 62.3^\circ$$

Thus: **R** = 226 N ⦣ 62.3°



PROBLEM 2.37

Knowing that $\alpha = 40°$, determine the resultant of the three forces shown.

SOLUTION

60-lb Force:

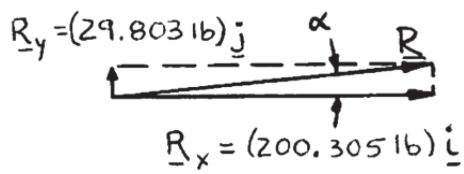
$$F_x = (60 \text{ lb})\cos 20° = 56.382 \text{ lb}$$
$$F_y = (60 \text{ lb})\sin 20° = 20.521 \text{ lb}$$

80-lb Force:

$$F_x = (80 \text{ lb})\cos 60° = 40.000 \text{ lb}$$
$$F_y = (80 \text{ lb})\sin 60° = 69.282 \text{ lb}$$

120-lb Force:

$$F_x = (120 \text{ lb})\cos 30° = 103.923 \text{ lb}$$
$$F_y = -(120 \text{ lb})\sin 30° = -60.000 \text{ lb}$$



and

$$R_x = \Sigma F_x = 200.305 \text{ lb}$$
$$R_y = \Sigma F_y = 29.803 \text{ lb}$$
$$R = \sqrt{(200.305 \text{ lb})^2 + (29.803 \text{ lb})^2}$$
$$= 202.510 \text{ lb}$$

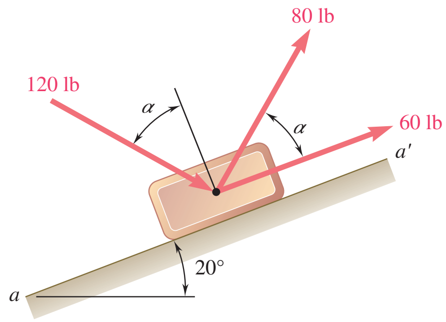
Further:

$$\tan\alpha = \frac{29.803}{200.305}$$

$$\alpha = \tan^{-1}\frac{29.803}{200.305}$$
$$= 8.46°$$

$\mathbf{R} = 203 \text{ lb} \measuredangle 8.46°$ ◀

PROBLEM 2.38



Knowing that $\alpha = 75°$, determine the resultant of the three forces shown.

SOLUTION

60-lb Force:

$$F_x = (60 \text{ lb})\cos 20° = 56.382 \text{ lb}$$
$$F_y = (60 \text{ lb})\sin 20° = 20.521 \text{ lb}$$

80-lb Force:

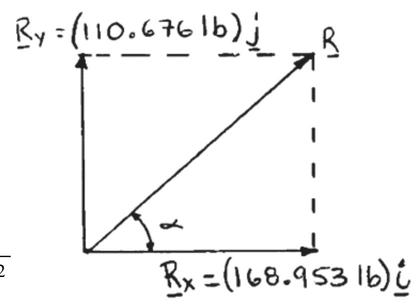
$$F_x = (80 \text{ lb})\cos 95° = -6.9725 \text{ lb}$$
$$F_y = (80 \text{ lb})\sin 95° = 79.696 \text{ lb}$$

120-lb Force:

$$F_x = (120 \text{ lb})\cos 5° = 119.543 \text{ lb}$$
$$F_y = (120 \text{ lb})\sin 5° = 10.459 \text{ lb}$$

Then

$$R_x = \Sigma F_x = 168.953 \text{ lb}$$
$$R_y = \Sigma F_y = 110.676 \text{ lb}$$



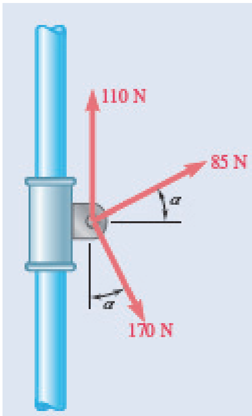
and

$$R = \sqrt{(168.953 \text{ lb})^2 + (110.676 \text{ lb})^2}$$
$$= 201.976 \text{ lb}$$

$$\tan\alpha = \frac{110.676}{168.953}$$
$$\tan\alpha = 0.65507$$
$$\alpha = 33.228°$$

$\mathbf{R} = 202 \text{ lb} \measuredangle 33.2°$ ◀

PROBLEM 2.39



A collar that can slide on a vertical rod is subjected to the three forces shown. Determine (*a*) the required value of α if the resultant of the three forces is to be horizontal, (*b*) the corresponding magnitude of the resultant.

SOLUTION

$$R_x = \Sigma F_x$$
$$= (85 \text{ N})\cos\alpha + (170 \text{ N})\sin(\alpha) \quad (1)$$

$$R_y = \Sigma F_y$$
$$= +(110 \text{ N}) + (85 \text{ N})\sin(\alpha) - (170 \text{ N})\cos\alpha \quad (2)$$

(*a*) For **R** to be horizontal, we must have $R_y = 0$. We make $R_y = 0$ in Eq. (2):

$$110 + 85\sin\alpha - 170\cos\alpha = 0$$
$$22 + 17\sin\alpha - 34\cos\alpha = 0$$
$$17\sin\alpha + 22 = -34\sqrt{1-\sin^2\alpha}$$
$$289\sin^2\alpha + 748\sin\alpha + 484 = 1156\left(1-\sin^2\alpha\right)$$
$$1445\sin^2\alpha + 748\sin\alpha - 672 = 0$$

Solving by use of the quadratic formula:

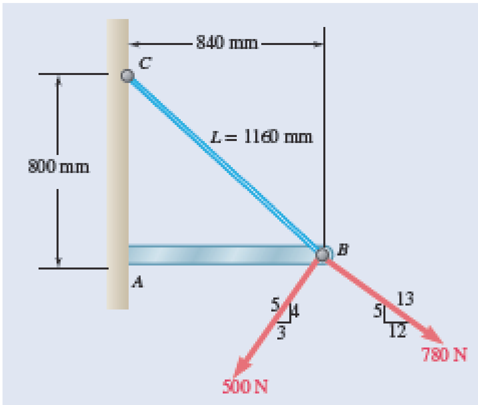
$$\sin\alpha = 0.47059$$
$$\alpha = 28.072°$$

$\alpha = 28.1°$ ◀

(*b*) Since $R = R_x$ using Eq. (1):

$$R = 85\cos 28.072° + 170\sin 28.072°$$
$$= 155.0 \text{ N}$$

$R = 155.0$ N ◀



PROBLEM 2.40

For the beam of Problem 2.36, determine (*a*) the required tension in cable *BC* if the resultant of the three forces exerted at Point *B* is to be vertical, (*b*) the corresponding magnitude of the resultant.

SOLUTION

$$R_x = \Sigma F_x = -\frac{840}{1160}T_{BC} + \frac{12}{13}(780\text{ N}) - \frac{3}{5}(500\text{ N})$$

$$R_x = -\frac{21}{29}T_{BC} + 420\text{ N} \qquad (1)$$

$$R_y = \Sigma F_y = \frac{800}{1160}T_{BC} - \frac{5}{13}(780\text{ N}) - \frac{4}{5}(500\text{ N})$$

$$R_y = \frac{20}{29}T_{BC} - 700\text{ N} \qquad (2)$$

(*a*) For **R** to be vertical, we must have $R_x = 0$

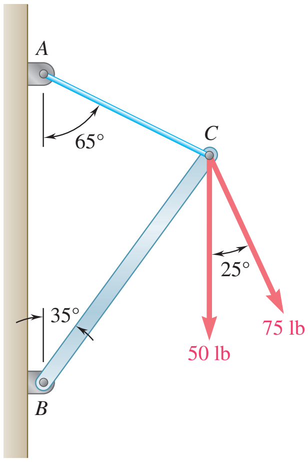
Set $R_x = 0$ in Eq. (1) $\quad -\frac{21}{29}T_{BC} + 420\text{ N} = 0 \quad$ $T_{BC} = 580\text{ N}$

(*b*) Substituting for T_{BC} in Eq. (2):

$$R_y = \frac{20}{29}(580\text{ N}) - 700\text{ N}$$

$$R_y = -300\text{ N}$$

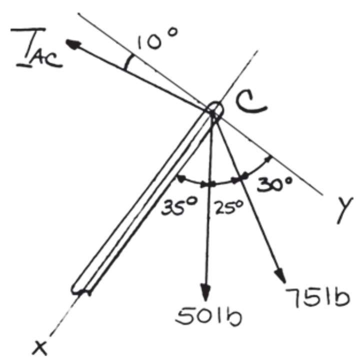
$$R = |R_y| = 300\text{ N} \qquad R = 300\text{ N}$$



PROBLEM 2.41

Determine (*a*) the required tension in cable *AC*, knowing that the resultant of the three forces exerted at Point *C* of boom *BC* must be directed along *BC*, (*b*) the corresponding magnitude of the resultant.

SOLUTION



Using the *x* and *y* axes shown:

$$R_x = \Sigma F_x = T_{AC}\sin 10° + (50 \text{ lb})\cos 35° + (75 \text{ lb})\cos 60°$$
$$= T_{AC}\sin 10° + 78.458 \text{ lb} \qquad (1)$$

$$R_y = \Sigma F_y = (50 \text{ lb})\sin 35° + (75 \text{ lb})\sin 60° - T_{AC}\cos 10°$$
$$R_y = 93.631 \text{ lb} - T_{AC}\cos 10° \qquad (2)$$

(*a*) Set $R_y = 0$ in Eq. (2):

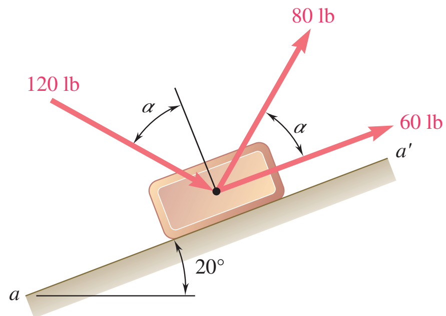
$$93.631 \text{ lb} - T_{AC}\cos 10° = 0$$
$$T_{AC} = 95.075 \text{ lb}$$

$T_{AC} = 95.1$ lb ◀

(*b*) Substituting for T_{AC} in Eq. (1):

$$R_x = (95.075 \text{ lb})\sin 10° + 78.458 \text{ lb}$$
$$= 94.968 \text{ lb}$$
$$R = R_x$$

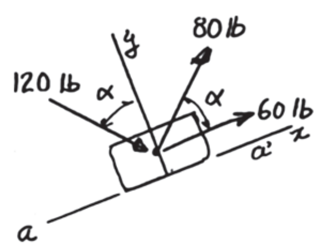
$R = 95.0$ lb ◀



PROBLEM 2.42

For the block of Problems 2.37 and 2.38, determine (*a*) the required value of α if the resultant of the three forces shown is to be parallel to the incline, (*b*) the corresponding magnitude of the resultant.

SOLUTION



Select the x axis to be along $a\,a'$.

Then

$$R_x = \Sigma F_x = (60\text{ lb}) + (80\text{ lb})\cos\alpha + (120\text{ lb})\sin\alpha \qquad (1)$$

and

$$R_y = \Sigma F_y = (80\text{ lb})\sin\alpha - (120\text{ lb})\cos\alpha \qquad (2)$$

(*a*) Set $R_y = 0$ in Eq. (2).

$$(80\text{ lb})\sin\alpha - (120\text{ lb})\cos\alpha = 0$$

Dividing each term by $\cos\alpha$ gives:

$$(80\text{ lb})\tan\alpha = 120\text{ lb}$$

$$\tan\alpha = \frac{120\text{ lb}}{80\text{ lb}}$$

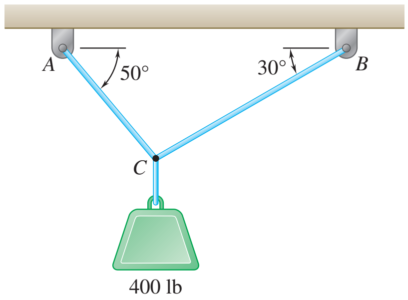
$$\alpha = 56.310°$$

$\alpha = 56.3°$ ◀

(*b*) Substituting for α in Eq. (1) gives:

$$R_x = 60\text{ lb} + (80\text{ lb})\cos 56.31° + (120\text{ lb})\sin 56.31° = 204.22\text{ lb}$$

$R_x = 204\text{ lb}$ ◀

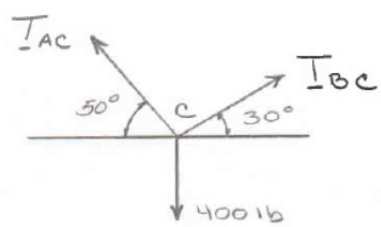


PROBLEM 2.43

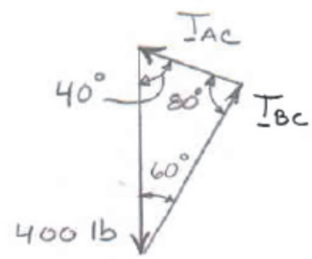
Two cables are tied together at *C* and are loaded as shown. Determine the tension (*a*) in cable *AC*, (*b*) in cable *BC*.

SOLUTION

Free-Body Diagram



Force Triangle

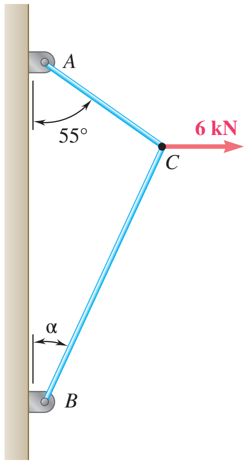


Law of sines:

$$\frac{T_{AC}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 40^\circ} = \frac{400 \text{ lb}}{\sin 80^\circ}$$

(*a*) $$T_{AC} = \frac{400 \text{ lb}}{\sin 80^\circ}(\sin 60^\circ)$$ $T_{AC} = 352 \text{ lb}$ ◀

(*b*) $$T_{BC} = \frac{400 \text{ lb}}{\sin 80^\circ}(\sin 40^\circ)$$ $T_{BC} = 261 \text{ lb}$ ◀

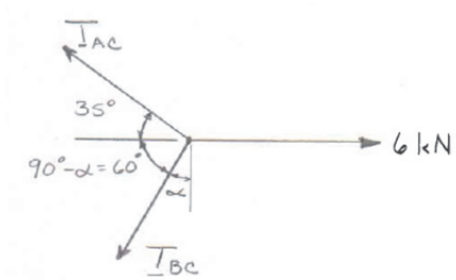


PROBLEM 2.44

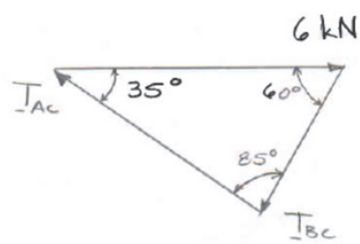
Two cables are tied together at C and are loaded as shown. Knowing that $\alpha = 30°$, determine the tension (*a*) in cable AC, (*b*) in cable BC.

SOLUTION

Free-Body Diagram



Force Triangle

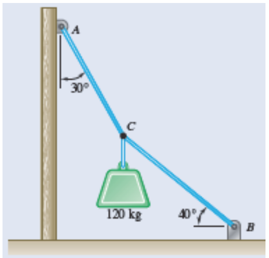


Law of sines:

$$\frac{T_{AC}}{\sin 60°} = \frac{T_{BC}}{\sin 35°} = \frac{6 \text{ kN}}{\sin 85°}$$

(*a*) $$T_{AC} = \frac{6 \text{ kN}}{\sin 85°}(\sin 60°)$$ $T_{AC} = 5.22$ kN ◀

(*b*) $$T_{BC} = \frac{6 \text{ kN}}{\sin 85°}(\sin 35°)$$ $T_{BC} = 3.45$ kN ◀

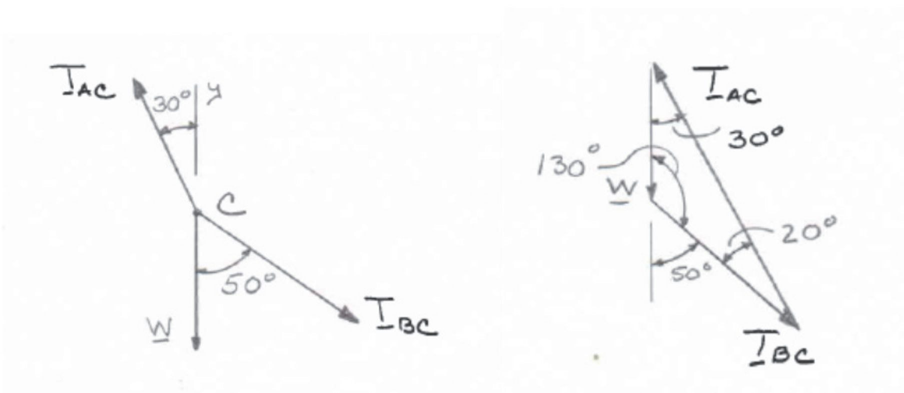


PROBLEM 2.45

Two cables are tied together at C and are loaded as shown. Determine the tension (*a*) in cable AC, (*b*) in cable BC.

SOLUTION

Free-Body Diagram **Force Triangle**



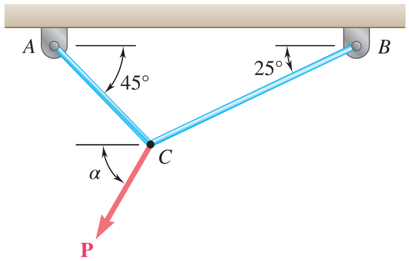
$$W = mg = (120 \text{ kg})(9.81 \text{ m/s}^2) = 1177 \text{ N}$$

Law of sines:

$$\frac{T_{AC}}{\sin 130°} = \frac{T_{BC}}{\sin 30°} = \frac{1.177 \text{ kN}}{\sin 20°}$$

(*a*) $$T_{AC} = \frac{1.177 \text{ kN}}{\sin 20°}(\sin 130°)$$ $T_{AC} = 2.64 \text{ kN}$ ◀

(*b*) $$T_{BC} = \frac{1.177 \text{ kN}}{\sin 20°}(\sin 30°)$$ $T_{BC} = 1.721 \text{ kN}$ ◀

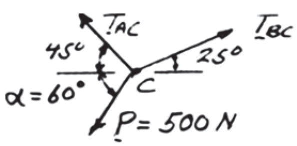


PROBLEM 2.46

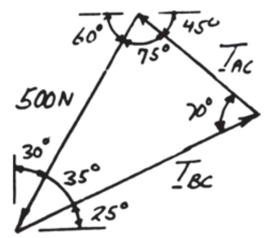
Two cables are tied together at C and are loaded as shown. Knowing that $\mathbf{P} = 500$ N and $\alpha = 60°$, determine the tension in (*a*) in cable AC, (*b*) in cable BC.

SOLUTION

Free-Body Diagram



Force Triangle

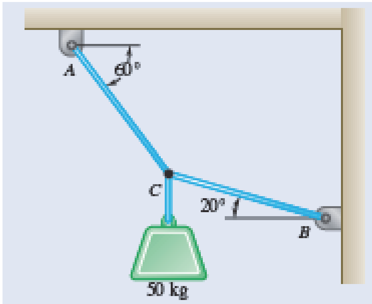


Law of sines: $$\frac{T_{AC}}{\sin 35°} = \frac{T_{BC}}{\sin 75°} = \frac{500 \text{ N}}{\sin 70°}$$

(*a*) $$T_{AC} = \frac{500 \text{ N}}{\sin 70°} \sin 35°$$ $T_{AC} = 305$ N ◀

(*b*) $$T_{BC} = \frac{500 \text{ N}}{\sin 70°} \sin 75°$$ $T_{BC} = 514$ N ◀

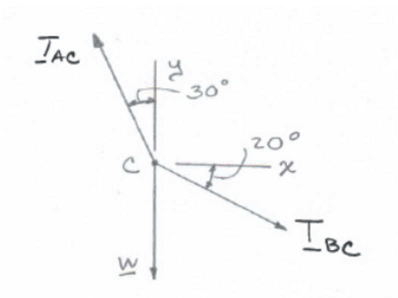
PROBLEM 2.47



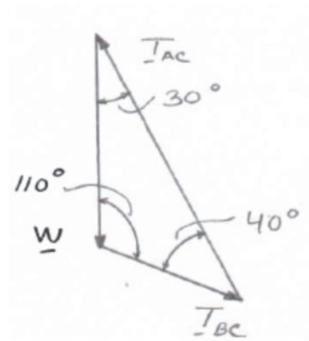
Two cables are tied together at *C* and are loaded as shown. Determine the tension (*a*) in cable *AC*, (*b*) in cable *BC*.

SOLUTION

Free-Body Diagram



Force Triangle

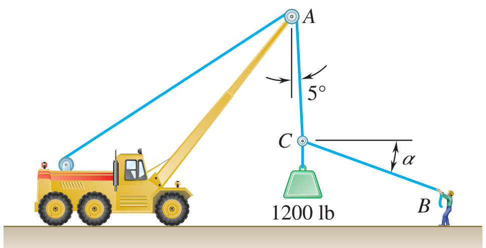


$$W = mg = (50\text{ kg})(9.81\text{ m/s}^2) = 490\text{ N}$$

Law of sines:
$$\frac{T_{AC}}{\sin 110°} = \frac{T_{BC}}{\sin 30°} = \frac{490\text{ N}}{\sin 40°}$$

(*a*)
$$T_{AC} = \frac{490\text{ N}}{\sin 40°}\sin 110°$$
$T_{AC} = 716$ N ◀

(*b*)
$$T_{BC} = \frac{490\text{ N}}{\sin 40°}\sin 30°$$
$T_{BC} = 381$ N ◀

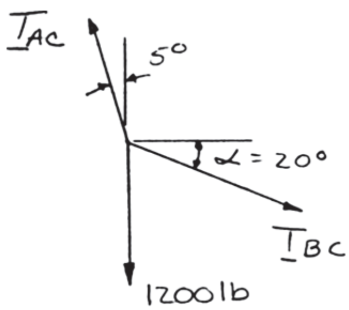


PROBLEM 2.48

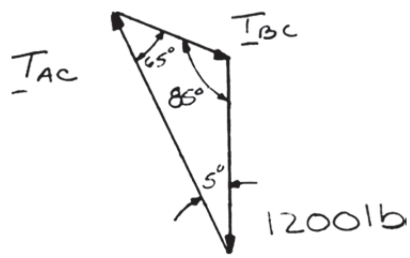
Knowing that $\alpha = 20°$, determine the tension (*a*) in cable *AC*, (*b*) in rope *BC*.

SOLUTION

Free-Body Diagram



Force Triangle



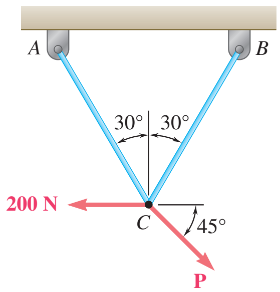
Law of sines:
$$\frac{T_{AC}}{\sin 110°} = \frac{T_{BC}}{\sin 5°} = \frac{1200 \text{ lb}}{\sin 65°}$$

(*a*)
$$T_{AC} = \frac{1200 \text{ lb}}{\sin 65°} \sin 110°$$

$T_{AC} = 1244$ lb ◀

(*b*)
$$T_{BC} = \frac{1200 \text{ lb}}{\sin 65°} \sin 5°$$

$T_{BC} = 115.4$ lb ◀

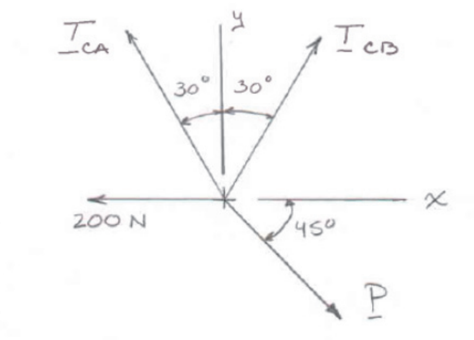


PROBLEM 2.49

Two cables are tied together at C and are loaded as shown. Knowing that $P = 300$ N, determine the tension in cables AC and BC.

SOLUTION

Free-Body Diagram



$\xrightarrow{+} \Sigma F_x = 0 \qquad -T_{CA}\sin 30° + T_{CB}\sin 30° - P\cos 45° - 200\text{N} = 0$

For $P = 200\text{N}$ we have,

$$-0.5T_{CA} + 0.5T_{CB} + 212.13 - 200 = 0 \ (1)$$

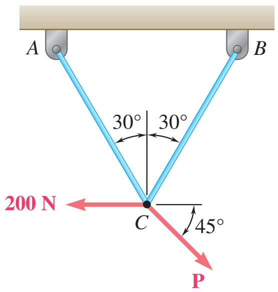
$+\uparrow \Sigma F_y = 0 \qquad T_{CA}\cos 30° - T_{CB}\cos 30° - P\sin 45° = 0$

$$0.86603T_{CA} + 0.86603T_{CB} - 212.13 = 0 \quad (2)$$

Solving equations (1) and (2) simultaneously gives, $T_{CA} = 134.6$ N ◀

$T_{CB} = 110.4$ N ◀

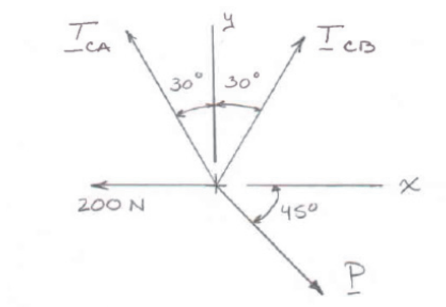
PROBLEM 2.50



Two cables are tied together at C and are loaded as shown. Determine the range of values of **P** for which both cables remain taut.

SOLUTION

Free-Body Diagram



$\xrightarrow{+} \Sigma F_x = 0 \qquad -T_{CA}\sin 30° + T_{CB}\sin 30° - P\cos 45° - 200\text{N} = 0$

For $T_{CA} = 0$ we have,

$$0.5T_{CB} + 0.70711P - 200 = 0 \quad (1)$$

$+\uparrow \Sigma F_y = 0 \qquad T_{CA}\cos 30° - T_{CB}\cos 30° - P\sin 45° = 0$; again setting $T_{CA} = 0$ yields,

$$0.86603T_{CB} - 0.70711P = 0 \quad (2)$$

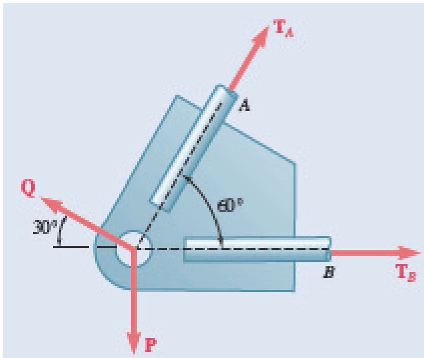
Adding equations (1) and (2) gives, $1.36603T_{CB} = 200$ hence $T_{CB} = 146.410\text{N}$ and $P = 179.315\text{N}$

Substituting for $T_{CB} = 0$ into the equilibrium equations and solving simultaneously gives,

$$-0.5T_{CA} + 0.70711P - 200 = 0$$
$$0.86603T_{CA} - 0.70711P = 0$$

And $T_{CA} = 546.40\text{N}$, $P = 669.20\text{N}$ Thus for both cables to remain taut, load P must be within the range of 179.315 N and 669.20 N.

$179.3 \text{ N} < P < 669 \text{ N}$ ◀

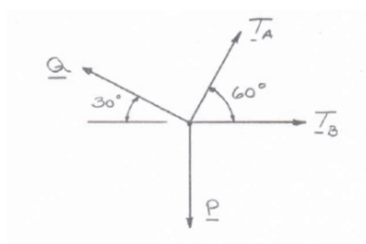


PROBLEM 2.51

Two forces **P** and **Q** are applied as shown to an aircraft connection. Knowing that the connection is in equilibrium and that P = 600 lb and Q = 800 lb, determine the tension in rods A and B.

SOLUTION

Free-Body Diagram



Resolving the forces into x- and y-directions:

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} + \mathbf{T}_A + \mathbf{T}_B = 0$$

Substituting components:

$$\mathbf{R} = -(600 \text{ lb})\mathbf{j} - [(800 \text{ lb})\cos 30°]\mathbf{i} + [(800 \text{ lb})\sin 30°]\mathbf{j} + T_B\mathbf{i} + (T_A \cos 60°)\mathbf{i} + (T_A \sin 60°)\mathbf{j} = 0$$

Summing forces in the y-direction:

$$-600 \text{ lb} + (800 \text{ lb})\sin 30° + T_A \sin 60° = 0$$

$$T_A = 230.94 \text{ lb} \qquad T_A = 231 \text{ lb} \blacktriangleleft$$

Summing forces in the x-direction:

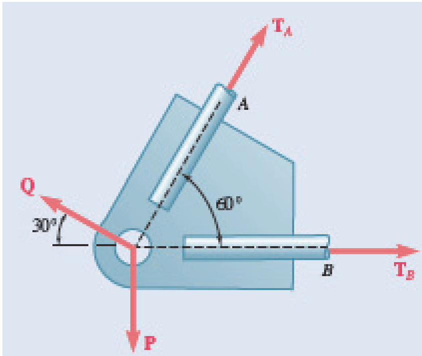
$$-(800 \text{ lb})\cos 30° + T_B + T_A \cos 60° = 0$$

Thus,

$$T_B = -(230.94 \text{ lb})\cos 60° + (800 \text{ lb})\cos 30°$$

$$= 577.35 \text{ lb} \qquad T_B = 577 \text{ lb} \blacktriangleleft$$

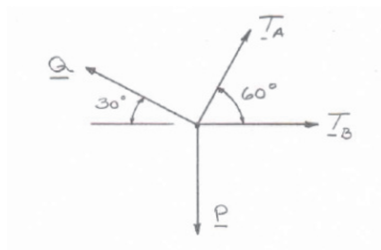
PROBLEM 2.52



Two forces **P** and **Q** are applied as shown to an aircraft connection. Knowing that the connection is in equilibrium and that the tensions in rods *A* and *B* are $T_A = 240$ lb and $T_B = 500$ lb, determine the magnitudes of **P** and **Q**.

SOLUTION

Free-Body Diagram



Resolving the forces into *x*- and *y*-directions:

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} + \mathbf{T}_A + \mathbf{T}_B = 0$$

Substituting components:

$$\begin{aligned} \mathbf{R} = & -P\mathbf{j} - Q\cos 30°\mathbf{i} + Q\sin 30°\mathbf{j} \\ & +[(240 \text{ lb})\cos 60°]\mathbf{i} \\ & +[(240 \text{ lb})\sin 60°]\mathbf{j} + (500 \text{ lb})\mathbf{i} \end{aligned}$$

Summing forces in the *x*-direction:

$$-Q\cos 30° + (240 \text{ lb})\cos 60° + 500 \text{ lb} = 0$$

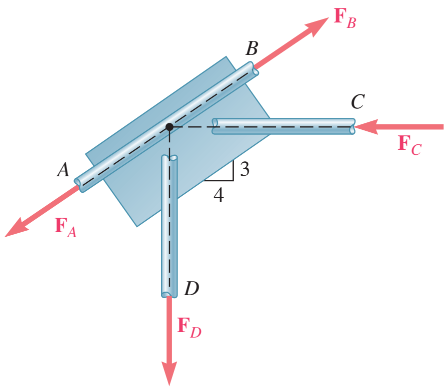
$$Q = 715.91 \text{ lb}$$

Summing forces in the *y*-direction:

$$-P + Q\sin 30° + (240 \text{ lb})\sin 60° = 0$$

$$\begin{aligned} P &= Q\sin 30° + (240 \text{ lb})\sin 60° \\ &= (715.91 \text{ lb})\sin 30° + (240 \text{ lb})\sin 60° \\ &= 565.80 \text{ lb} \end{aligned}$$

$P = 566$ lb; $Q = 716$ lb ◀

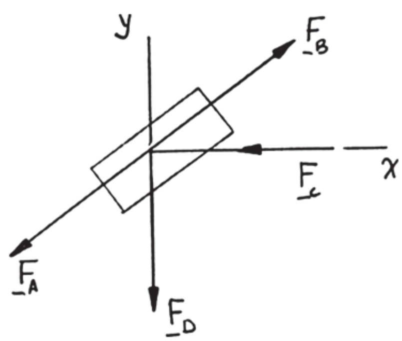


PROBLEM 2.53

A welded connection is in equilibrium under the action of the four forces shown. Knowing that $F_A = 8$ kN and $F_B = 16$ kN, determine the magnitudes of the other two forces.

SOLUTION

Free-Body Diagram of Connection



$$\Sigma F_x = 0: \quad \frac{3}{5}F_B - F_C - \frac{3}{5}F_A = 0$$

With $F_A = 8$ kN

$F_B = 16$ kN

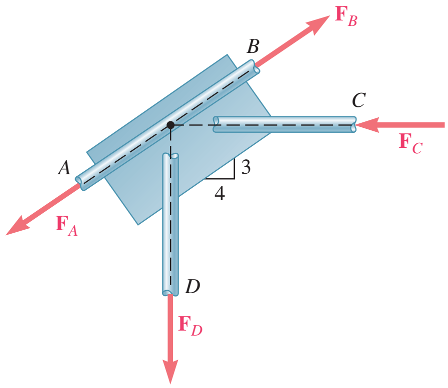
$$F_C = \frac{4}{5}(16 \text{ kN}) - \frac{4}{5}(8 \text{ kN})$$

$F_C = 6.40$ kN ◀

$$\Sigma F_y = 0: \quad -F_D + \frac{3}{5}F_B - \frac{3}{5}F_A = 0$$

With F_A and F_B as above: $F_D = \frac{3}{5}(16 \text{ kN}) - \frac{3}{5}(8 \text{ kN})$

$F_D = 4.80$ kN ◀

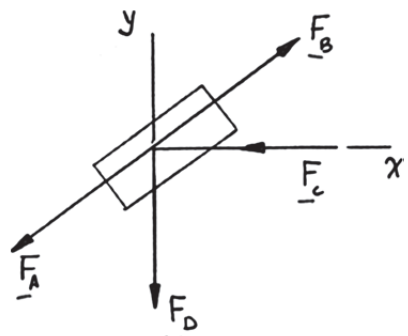


PROBLEM 2.54

A welded connection is in equilibrium under the action of the four forces shown. Knowing that $F_A = 5$ kN and $F_D = 6$ kN, determine the magnitudes of the other two forces.

SOLUTION

Free-Body Diagram of Connection



$$\Sigma F_y = 0: \quad -F_D - \frac{3}{5}F_A + \frac{3}{5}F_B = 0$$

or

$$F_B = F_D + \frac{3}{5}F_A$$

With

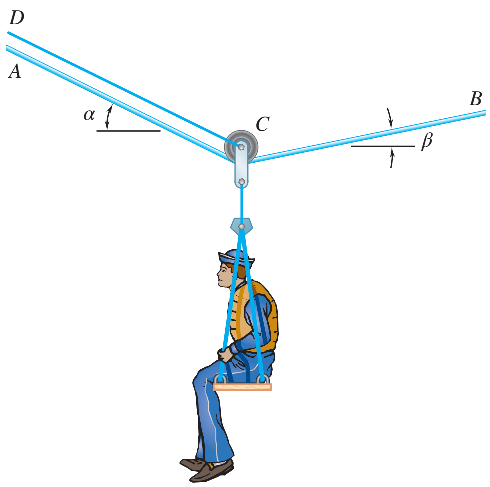
$$F_A = 5 \text{ kN}, \quad F_D = 8 \text{ kN}$$

$$F_B = \frac{5}{3}\left[6 \text{ kN} + \frac{3}{5}(5 \text{ kN})\right] \qquad F_B = 15.00 \text{ kN} \blacktriangleleft$$

$$\Sigma F_x = 0: \quad -F_C + \frac{4}{5}F_B - \frac{4}{5}F_A = 0$$

$$F_C = \frac{4}{5}(F_B - F_A)$$

$$= \frac{4}{5}(15 \text{ kN} - 5 \text{ kN}) \qquad F_C = 8.00 \text{ kN} \blacktriangleleft$$

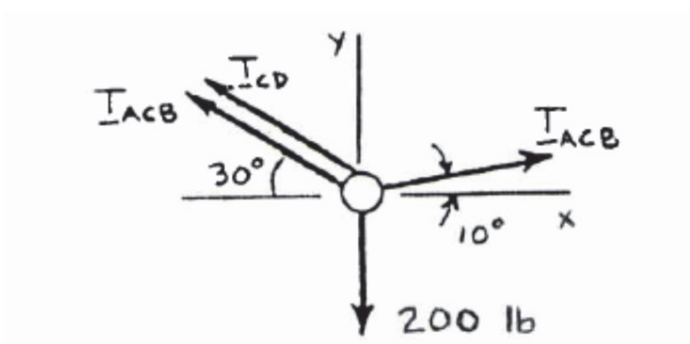


PROBLEM 2.55

A sailor is being rescued using a boatswain's chair that is suspended from a pulley that can roll freely on the support cable *ACB* and is pulled at a constant speed by cable *CD*. Knowing that $\alpha = 30°$ and $\beta = 10°$ and that the combined weight of the boatswain's chair and the sailor is 200 lb, determine the tension (*a*) in the support cable *ACB*, (*b*) in the traction cable *CD*.

SOLUTION

Free-Body Diagram



$$\xrightarrow{+} \Sigma F_x = 0: \quad T_{ACB} \cos 10° - T_{ACB} \cos 30° - T_{CD} \cos 30° = 0$$

$$T_{CD} = 0.137158 T_{ACB} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad T_{ACB} \sin 10° + T_{ACB} \sin 30° + T_{CD} \sin 30° - 200 = 0$$

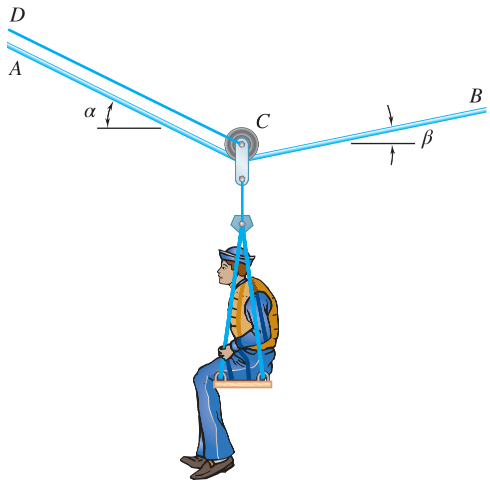
$$0.67365 T_{ACB} + 0.5 T_{CD} = 200 \qquad (2)$$

(*a*) Substitute (1) into (2): $0.67365 T_{ACB} + 0.5(0.137158 T_{ACB}) = 200$

$T_{ACB} = 269.46$ lb $\qquad T_{ACB} = 269$ lb ◀

(*b*) From (1): $T_{CD} = 0.137158(269.46 \text{ lb})$ $\qquad T_{CD} = 37.0$ lb ◀

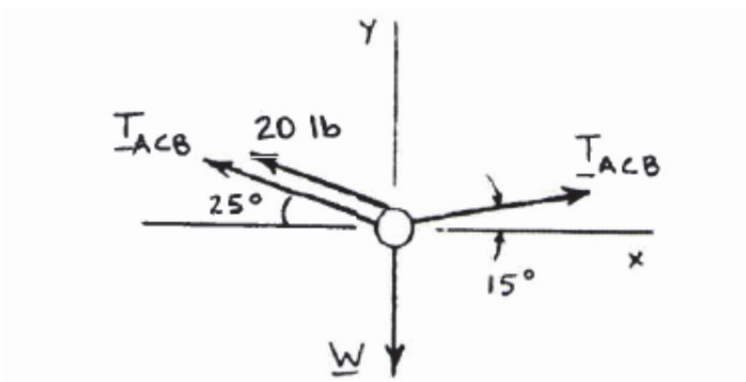
PROBLEM 2.56



A sailor is being rescued using a boatswain's chair that is suspended from a pulley that can roll freely on the support cable *ACB* and is pulled at a constant speed by cable *CD*. Knowing that $\alpha = 25°$ and $\beta = 15°$ and that the tension in cable *CD* is 20 lb, determine (*a*) the combined weight of the boatswain's chair and the sailor, (*b*) the tension in the support cable *ACB*.

SOLUTION

Free-Body Diagram



$$\xrightarrow{+} \Sigma F_x = 0: \quad T_{ACB}\cos 15° - T_{ACB}\cos 25° - (20 \text{ lb})\cos 25° = 0$$

$$T_{ACB} = 304.04 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad (304.04 \text{ lb})\sin 15° + (304.04 \text{ lb})\sin 25°$$

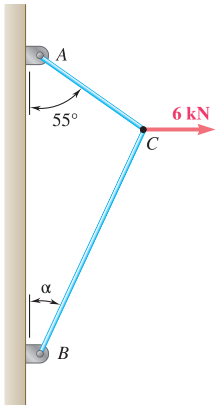
$$+(20 \text{ lb})\sin 25° - W = 0$$

$$W = 215.64 \text{ lb}$$

(*a*) $W = 216$ lb ◀

(*b*) $T_{ACB} = 304$ lb ◀

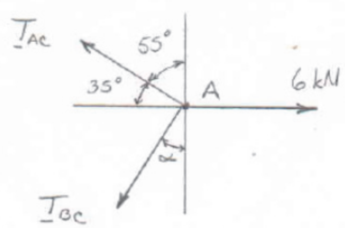
PROBLEM 2.57



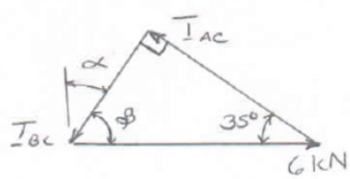
For the cables of prob. 2.44, find the value of α for which the tension is as small as possible (*a*) in cable *bc*, (*b*) in both cables simultaneously. In each case determine the tension in each cable.

SOLUTION

Free-Body Diagram



Force Triangle



(*a*) For a minimum tension in cable *BC,* set angle between cables to 90 degrees.

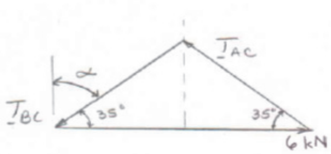
By inspection, $\alpha = 35.0° ◀$

$$T_{AC} = (6 \text{ kN})\cos 35° \qquad T_{AC} = 4.91 \text{ kN} ◀$$

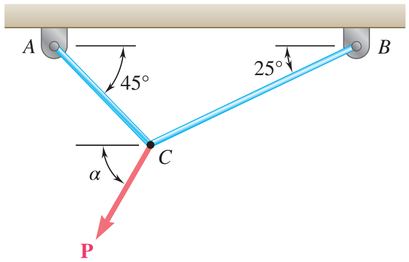
$$T_{BC} = (6 \text{ kN})\sin 35° \qquad T_{BC} = 3.44 \text{ kN} ◀$$

(*b*) For equal tension in both cables, the force triangle will be an isosceles.

Therefore, by inspection, $\alpha = 55.0° ◀$



$$T_{AC} = T_{BC} = (1/2)\frac{6 \text{ kN}}{\cos 35°} \qquad T_{AC} = T_{BC} = 3.66 \text{ kN} ◀$$

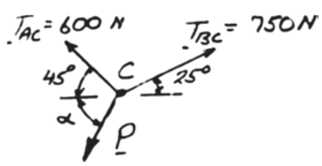


PROBLEM 2.58

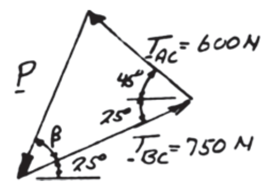
For the cables of Problem 2.46, it is known that the maximum allowable tension is 600 N in cable *AC* and 750 N in cable *BC*. Determine (*a*) the maximum force **P** that can be applied at *C*, (*b*) the corresponding value of α.

SOLUTION

Free-Body Diagram



Force Triangle

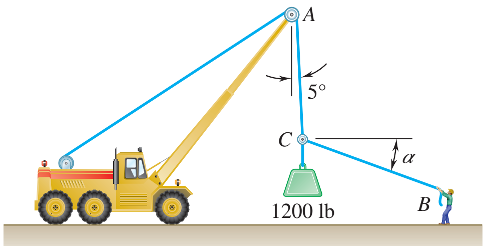


(*a*) Law of cosines $\quad P^2 = (600)^2 + (750)^2 - 2(600)(750)\cos(25° + 45°)$

$P = 784.02 \text{ N}$ $\quad$ $P = 784 \text{ N}$ ◀

(*b*) Law of sines $\quad \dfrac{\sin\beta}{600 \text{ N}} = \dfrac{\sin(25° + 45°)}{784.02 \text{ N}}$

$\beta = 46.0° \quad \therefore \quad \alpha = 46.0° + 25°$ $\quad$ $\alpha = 71.0°$ ◀

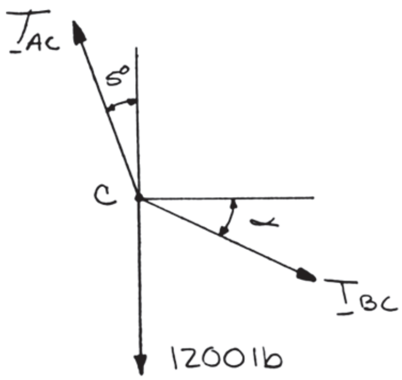


PROBLEM 2.59

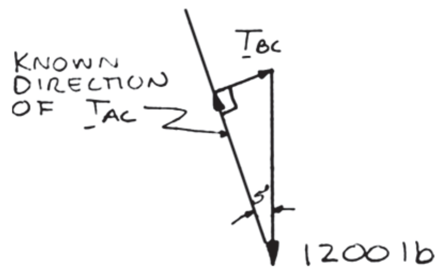
For the situation described in Figure P2.48, determine (*a*) the value of α for which the tension in rope *BC* is as small as possible, (*b*) the corresponding value of the tension.

SOLUTION

Free-Body Diagram



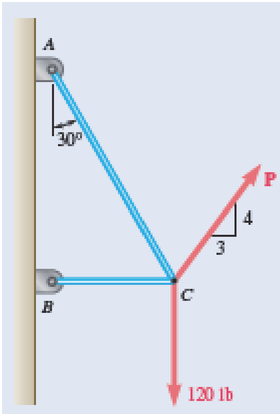
Force Triangle



To be smallest, T_{BC} must be perpendicular to the direction of T_{AC}.

(*a*) Thus, $\alpha = 5.00°$ $\qquad$ $\alpha = 5.00°$ ◀

(*b*) $T_{BC} = (1200 \text{ lb})\sin 5°$ $\qquad$ $T_{BC} = 104.6 \text{ lb}$ ◀

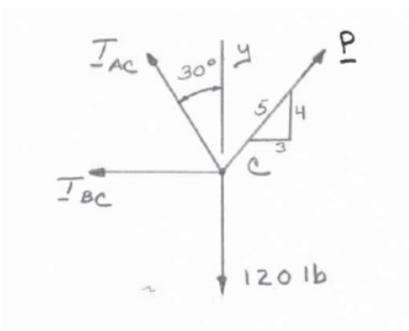


PROBLEM 2.60

Two cables tied together at C are loaded as shown. Determine the range of values of P for which both cables remain taut.

SOLUTION

Free-Body Diagram



$$\Sigma F_x = 0: \quad \frac{3}{5}P - T_{BC} - T_{AC}\sin 30° = 0$$

$$P = \frac{5}{3}\left(T_{BC} + T_{AC}\sin 30°\right) \qquad (1)$$

$$\Sigma F_y = 0: \quad \frac{4}{5}P + T_{AC}\cos 30° - 120 = 0$$

$$P = \frac{5}{4}\left(120 - T_{AC}\cos 30°\right) \qquad (2)$$

Requirement: $T_{AC} = 0$:

From Eq. (2): $P = 150.0$ lb

Requirement: $T_{BC} = 0$:

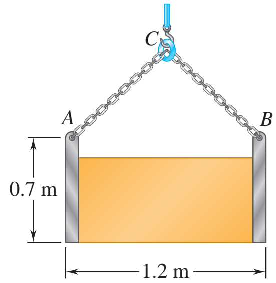
From Eq. (1): $P = \frac{5}{3}\left(T_{AC}\sin 30°\right)$

From Eq. (2): $P = \frac{5}{4}\left(120 - T_{AC}\cos 30°\right)$

Solving simultaneously yields:

$T_{AC} = 78.294$ lb and $P = 65.2$ lb

$$65.2 \text{ lb} < P < 150.0 \text{ lb} \blacktriangleleft$$

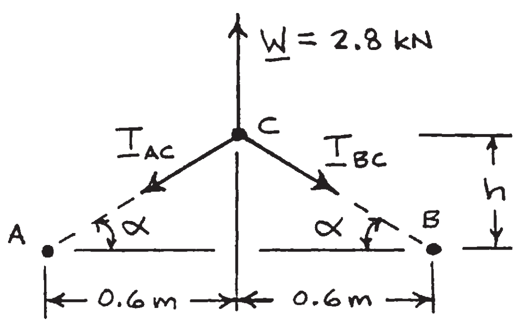


PROBLEM 2.61

A movable bin and its contents have a combined weight of 2.8 kN. Determine the shortest chain sling *ACB* that can be used to lift the loaded bin if the tension in the chain is not to exceed 5 kN.

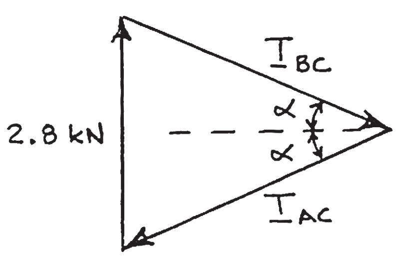
SOLUTION

Free-Body Diagram



$$\tan\alpha = \frac{h}{0.6\text{ m}} \quad (1)$$

Isosceles Force Triangle



Law of sines:

$$\sin\alpha = \frac{\frac{1}{2}(2.8\text{ kN})}{T_{AC}}$$

$$T_{AC} = 5\text{ kN}$$

$$\sin\alpha = \frac{\frac{1}{2}(2.8\text{ kN})}{5\text{ kN}}$$

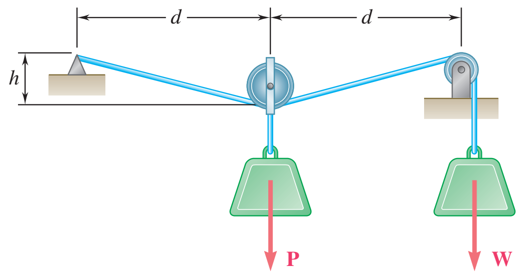
$$\alpha = 16.2602°$$

From Eq. (1): $\tan 16.2602° = \dfrac{h}{0.6\text{ m}}$ $\quad\therefore\quad h = 0.175000\text{ m}$

Half-length of chain $= AC = \sqrt{(0.6\text{ m})^2 + (0.175\text{ m})^2}$

$= 0.625\text{ m}$

Total length: $= 2 \times 0.625\text{ m}$ $\qquad$ 1.250 m ◀

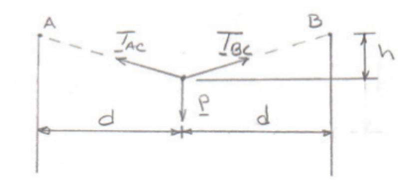


PROBLEM 2.62

For $W = 800$ N, $P = 200$ N, and $d = 600$ mm, determine the value of h consistent with equilibrium.

SOLUTION

Free-Body Diagram



$$T_{AC} = T_{BC} = 800 \text{ N}$$

$$AC = BC = \sqrt{(h^2 + d^2)}$$

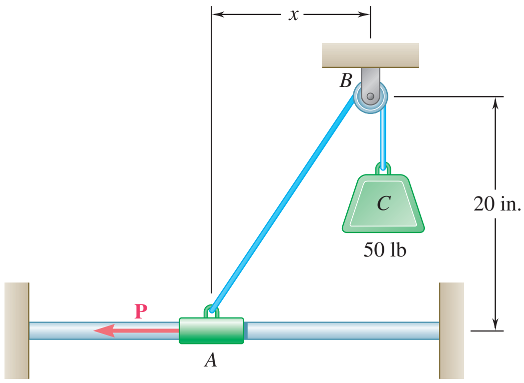
$$\Sigma F_y = 0: \quad 2(800 \text{ N})\frac{h}{\sqrt{h^2 + d^2}} - P = 0$$

$$800 = \frac{P}{2}\sqrt{1 + \left(\frac{d}{h}\right)^2}$$

Data: $P = 200$ N, $d = 600$ mm and solving for h

$$800 \text{ N} = \frac{200 \text{ N}}{2}\sqrt{1 + \left(\frac{600 \text{ mm}}{h}\right)^2}$$

$h = 75.6$ mm ◀

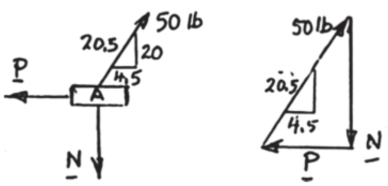


PROBLEM 2.63

Collar *A* is connected as shown to a 50-lb load and can slide on a frictionless horizontal rod. Determine the magnitude of the force **P** required to maintain the equilibrium of the collar when (*a*) $x = 4.5$ in., (*b*) $x = 15$ in.

SOLUTION

(*a*) **Free Body: Collar *A***

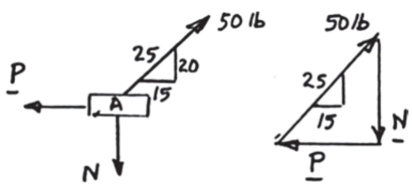


Force Triangle

$$\frac{P}{4.5} = \frac{50 \text{ lb}}{20.5}$$

$P = 10.98$ lb ◀

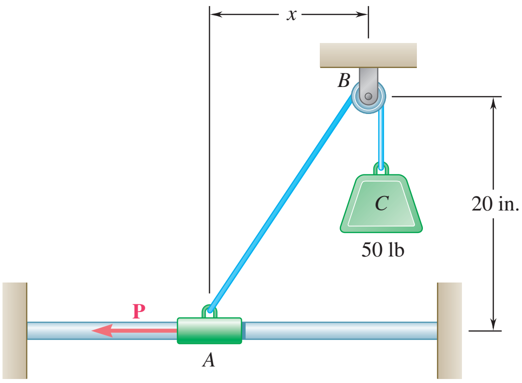
(*b*) **Free Body: Collar *A***



Force Triangle

$$\frac{P}{15} = \frac{50 \text{ lb}}{25}$$

$P = 30.0$ lb ◀

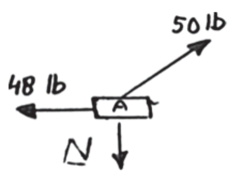


PROBLEM 2.64

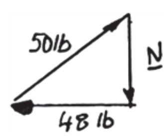
Collar A is connected as shown to a 50-lb load and can slide on a frictionless horizontal rod. Determine the distance x for which the collar is in equilibrium when $P = 48$ lb.

SOLUTION

Free Body: Collar A

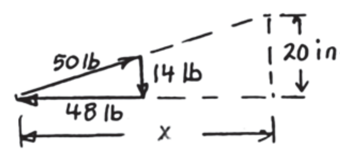


Force Triangle



$$N^2 = (50)^2 - (48)^2 = 196$$
$$N = 14.00 \text{ lb}$$

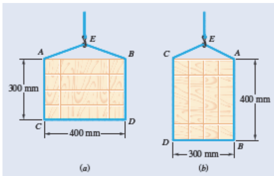
Similar Triangles



$$\frac{x}{20 \text{ in.}} = \frac{48 \text{ lb}}{14 \text{ lb}}$$

$x = 68.6$ in. ◀

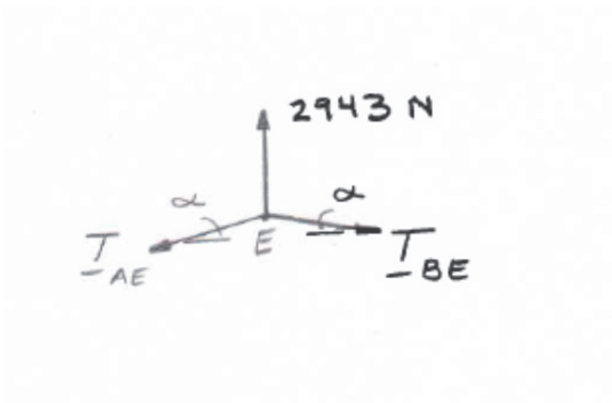
PROBLEM 2.65

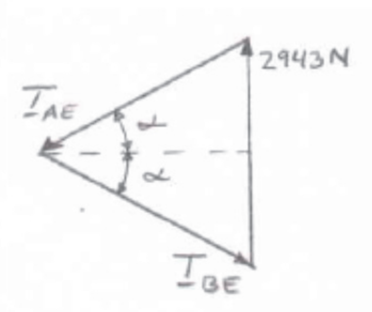


A cable loop of length 1.5 m is placed around a crate. Knowing that the mass of the crate is 300 kg, determine the tension in the cable for each of the arrangements shown.

SOLUTION

Free-Body Diagram **Isosceles Force Triangle**





$$W = (300 \text{ kg})(9.81 \text{ m/s}^2) = 2943.0 \text{ N}$$

$$EB = \frac{1}{2}(1500 \text{ mm} - 400 \text{ mm} - 300 \text{ mm} - 300 \text{ mm})$$

$$EB = 250 \text{ mm}$$

$$\alpha = \cos^{-1}\left(\frac{200 \text{ mm}}{250 \text{ mm}}\right) = 36.87°$$

$$T_{AE} = T_{BE}$$

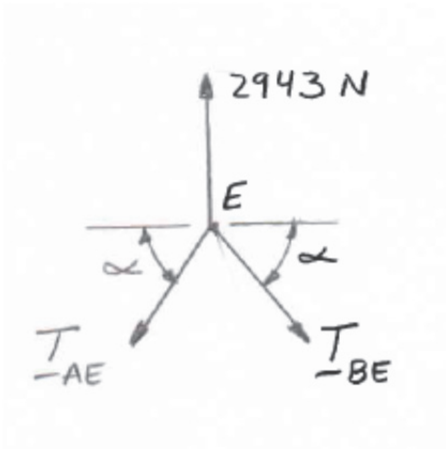
$$T_{AE} \sin\alpha = \frac{1}{2}(2943.0 \text{ N})$$

$$T_{AE} \sin 36.87° = \frac{1}{2}(2943.0 \text{ N})$$

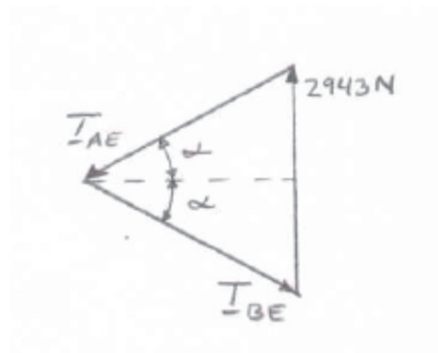
$$T_{AE} = 2452.5 \text{ N}$$

(*a*) $T_{AE} = 2450 \text{ N}$ ◀

Problem 2.65 (Continued)



Free-Body Diagram

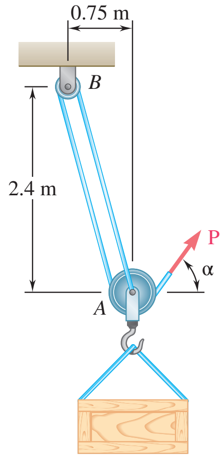


Isosceles Force Triangle

$$EB = \frac{1}{2}(1500 \text{ mm} - 300 \text{ mm} - 400 \text{ mm} - 400 \text{ mm})$$
$$EB = 250 \text{ mm}$$
$$\alpha = \cos^{-1}\left(\frac{150 \text{ mm}}{200 \text{ mm}}\right) = 41.41°$$

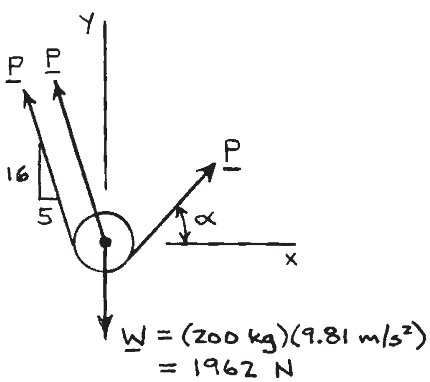
$$T_{AE} = T_{BE}$$
$$T_{AE} \sin\alpha = \frac{1}{2}(2943.0 \text{ N})$$
$$T_{AE} \sin 41.41° = \frac{1}{2}(2943.0 \text{ N})$$
$$T_{AE} = 2224.7 \text{ N}$$

(*b*) $T_{AE} = 2220 \text{ N}$ ◀

	**PROBLEM 2.66** A 200-kg crate is to be supported by the rope-and-pulley arrangement shown. Determine the magnitude and direction of the force **P** that must be exerted on the free end of the rope to maintain equilibrium. (*Hint:* The tension in the rope is the same on each side of a simple pulley. This can be proved by the methods of Ch. 4.)

SOLUTION

Free-Body Diagram: Pulley *A*



$$\xrightarrow{+} \Sigma F_x = 0: \quad -2P\left(\frac{5}{\sqrt{281}}\right) + P\cos\alpha = 0$$

$$\cos\alpha = 0.59655$$

$$\alpha = \pm 53.377°$$

For $\alpha = +53.377°$:

$$+\uparrow \Sigma F_y = 0: \quad 2P\left(\frac{16}{\sqrt{281}}\right) + P\sin 53.377° - 1962 \text{ N} = 0$$

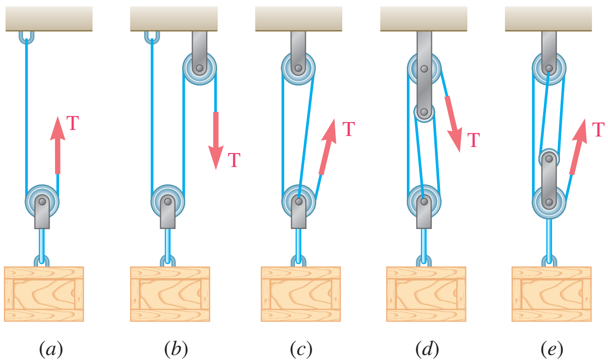
P = 724 N ∡53.4° ◀

For $\alpha = -53.377°$:

$$+\uparrow \Sigma F_y = 0: \quad 2P\left(\frac{16}{\sqrt{281}}\right) + P\sin(-53.377°) - 1962 \text{ N} = 0$$

P = 1773 ⦬53.4° ◀

PROBLEM 2.67

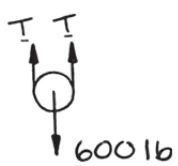


A 600-lb crate is supported by several rope-and-pulley arrangements as shown. Determine for each arrangement the tension in the rope. (See the hint for Problem 2.66.)

SOLUTION

Free-Body Diagram of Pulley

(*a*)

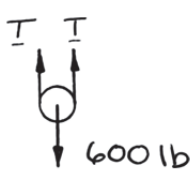


$$+\uparrow \Sigma F_y = 0: \quad 2T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{2}(600 \text{ lb})$$

$T = 300$ lb ◀

(*b*)

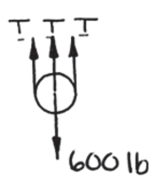


$$+\uparrow \Sigma F_y = 0: \quad 2T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{2}(600 \text{ lb})$$

$T = 300$ lb ◀

(*c*)

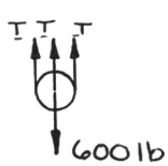


$$+\uparrow \Sigma F_y = 0: \quad 3T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{3}(600 \text{ lb})$$

$T = 200$ lb ◀

(*d*)



$$+\uparrow \Sigma F_y = 0: \quad 3T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{3}(600 \text{ lb})$$

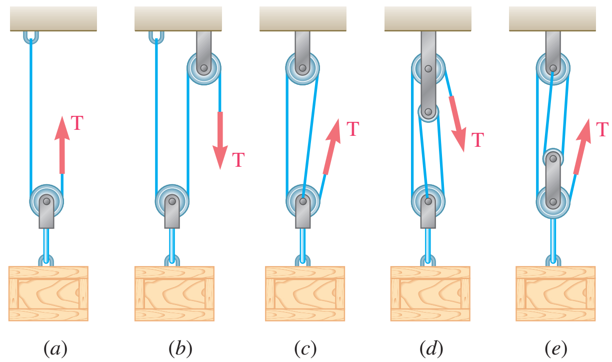
$T = 200$ lb ◀

(*e*)

$$+\uparrow \Sigma F_y = 0: \quad 4T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{4}(600 \text{ lb})$$

$T = 150.0$ lb ◀



PROBLEM 2.68

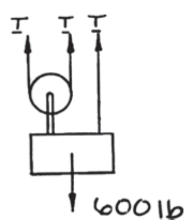
Solve Parts *b* and *d* of Problem 2.67, assuming that the free end of the rope is attached to the crate.

PROBLEM 2.67 A 600-lb crate is supported by several rope-and-pulley arrangements as shown. Determine for each arrangement the tension in the rope. (See the hint for Problem 2.66.)

SOLUTION

Free-Body Diagram of Pulley and Crate

(*b*)

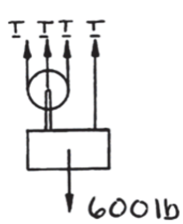


$$+\uparrow \Sigma F_y = 0: \quad 3T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{3}(600 \text{ lb})$$

$T = 200$ lb ◀

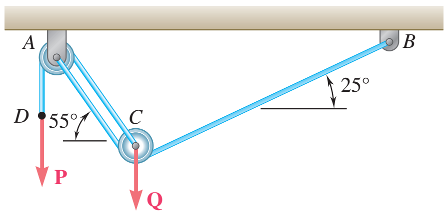
(*d*)



$$+\uparrow \Sigma F_y = 0: \quad 4T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{4}(600 \text{ lb})$$

$T = 150.0$ lb ◀

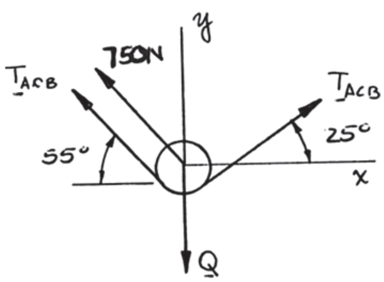


PROBLEM 2.69

A load **Q** is applied to the pulley *C*, which can roll on the cable *ACB*. The pulley is held in the position shown by a second cable *CAD*, which passes over the pulley *A* and supports a load **P**. Knowing that $P = 750$ N, determine (*a*) the tension in cable *ACB*, (*b*) the magnitude of load **Q**.

SOLUTION

Free-Body Diagram: Pulley *C*



(*a*) $\xrightarrow{+} \Sigma F_x = 0\text{:} \quad T_{ACB}(\cos 25° - \cos 55°) - (750 \text{ N})\cos 55° = 0$

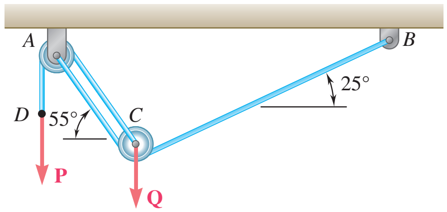
Hence: $T_{ACB} = 1292.88$ N

$T_{ACB} = 1293$ N ◀

(*b*) $+\uparrow \Sigma F_y = 0\text{:} \quad T_{ACB}(\sin 25° + \sin 55°) + (750 \text{ N})\sin 55° - Q = 0$

$$(1292.88 \text{ N})(\sin 25° + \sin 55°) + (750 \text{ N})\sin 55° - Q = 0$$

or $Q = 2219.8$ N $\quad Q = 2220$ N ◀

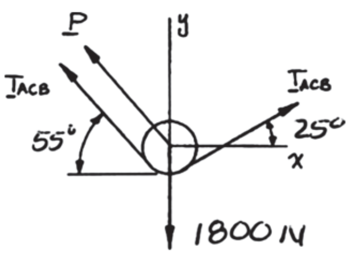


PROBLEM 2.70

An 1800-N load **Q** is applied to the pulley *C*, which can roll on the cable *ACB*. The pulley is held in the position shown by a second cable *CAD*, which passes over the pulley *A* and supports a load **P**. Determine (*a*) the tension in cable *ACB*, (*b*) the magnitude of load **P**.

SOLUTION

Free-Body Diagram: Pulley *C*



$$\xrightarrow{+} \Sigma F_x = 0: \quad T_{ACB}(\cos 25° - \cos 55°) - P\cos 55° = 0$$

or
$$P = 0.58010\, T_{ACB} \quad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad T_{ACB}(\sin 25° + \sin 55°) + P\sin 55° - 1800 \text{ N} = 0$$

or
$$1.24177 T_{ACB} + 0.81915 P = 1800 \text{ N} \quad (2)$$

(*a*) Substitute Equation (1) into Equation (2):

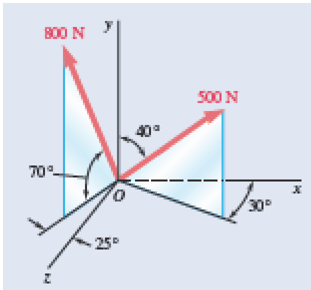
$$1.24177 T_{ACB} + 0.81915(0.58010 T_{ACB}) = 1800 \text{ N}$$

Hence:
$$T_{ACB} = 1048.37 \text{ N}$$

$$T_{ACB} = 1048 \text{ N} ◀$$

(*b*) Using (1), $P = 0.58010(1048.37 \text{ N}) = 608.16 \text{ N}$

$$P = 608 \text{ N} ◀$$



PROBLEM 2.71

Determine (*a*) the *x*, *y*, and *z* components of the 500-N force, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

SOLUTION

(*a*)

$$F_x = (500\text{ N})\sin 40°\cos 30°$$

$$F_x = 278.34\text{ N} \qquad F_x = 278\text{ N} ◀$$

$$F_y = (500\text{ N})\cos 40°$$

$$F_y = 383.02\text{ N} \qquad F_y = 383\text{ N} ◀$$

$$F_z = (500\text{ N})\sin 40°\sin 30°$$

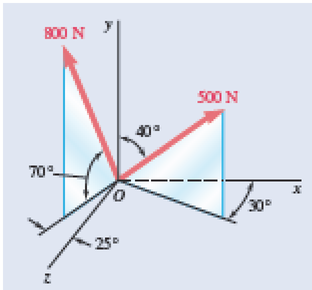
$$F_z = 160.697\text{ N} \qquad F_z = 160.7\text{ N} ◀$$

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{278.34\text{ N}}{500\text{ N}} \qquad \theta_x = 56.2° ◀$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{383.02\text{ N}}{500\text{ N}} \qquad \theta_y = 40.0° ◀$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{160.697\text{ N}}{500\text{ N}} \qquad \theta_z = 71.3° ◀$$



PROBLEM 2.72

Determine (*a*) the *x*, *y*, and *z* components of the 800-N force, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

SOLUTION

(*a*)

$$F_x = -(800\text{ N})\cos 70°\sin 25°$$

$$F_x = -115.635\text{ N} \qquad F_x = -115.6\text{ N} ◀$$

$$F_y = (800\text{ N})\sin\ 70°$$

$$F_y = 751.75\text{ N} \qquad F_y = 752\text{ N} ◀$$

$$F_z = (800\text{ N})\cos 70°\cos 25°$$

$$F_z = 247.98\text{ N} \qquad F_z = 248\text{ N} ◀$$

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{-115.635\text{ N}}{800\text{ N}} \qquad \theta_x = 98.3° ◀$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{751.75\text{ N}}{800\text{ N}} \qquad \theta_y = 20.0° ◀$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{247.98\text{ N}}{800\text{ N}} \qquad \theta_z = 71.9° ◀$$

Note: From the given data, we could have computed directly
$\theta_y = 90° - 35° = 55°$, which checks with the answer obtained.

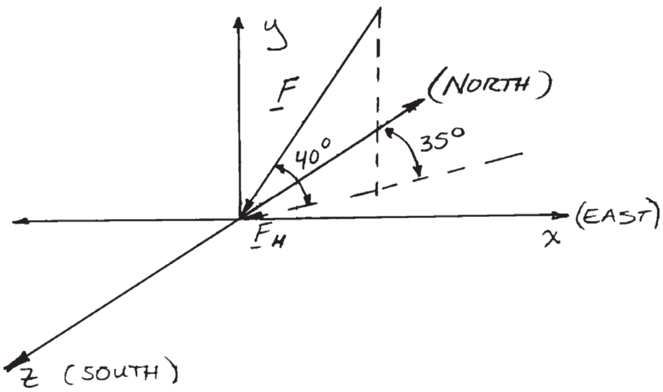
PROBLEM 2.73

A gun is aimed at a point A located 35° east of north. Knowing that the barrel of the gun forms an angle of 40° with the horizontal and that the maximum recoil force is 400 N, determine (*a*) the x, y, and z components of that force, (*b*) the values of the angles θ_x, θ_y, and θ_z defining the direction of the recoil force. (Assume that the x, y, and z axes are directed, respectively, east, up, and south.)

SOLUTION

Recoil force $F = 400 \text{ N}$

$$\therefore \quad F_H = (400 \text{ N})\cos 40° = 306.42 \text{ N}$$



(*a*)

$$F_x = -F_H \sin 35° = -(306.42 \text{ N})\sin 35° = -175.755 \text{ N}$$

$F_x = -175.8 \text{ N}$ ◀

$$F_y = -F \sin 40° = -(400 \text{ N})\sin 40° = -257.12 \text{ N}$$

$F_y = -257 \text{ N}$ ◀

$$F_z = +F_H \cos 35° = +(306.42 \text{ N})\cos 35° = +251.00 \text{ N}$$

$F_z = +251 \text{ N}$ ◀

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{-175.755 \text{ N}}{400 \text{ N}}$$

$\theta_x = 116.1°$ ◀

$$\cos\theta_y = \frac{F_y}{F} = \frac{-257.12 \text{ N}}{400 \text{ N}}$$

$\theta_y = 130.0°$ ◀

$$\cos\theta_z = \frac{F_z}{F} = \frac{251.00 \text{ N}}{400 \text{ N}}$$

$\theta_z = 51.1°$ ◀

PROBLEM 2.74

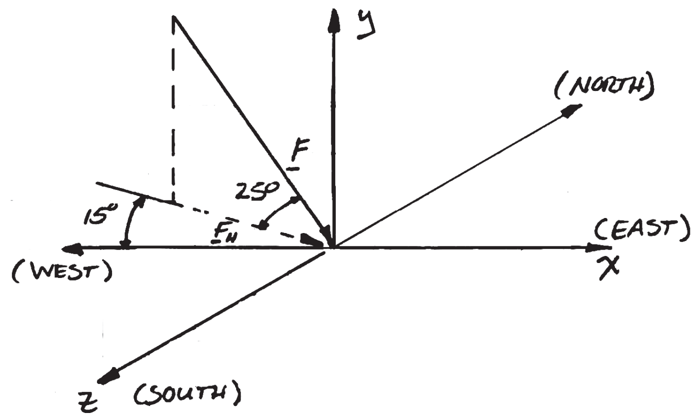
Solve Problem 2.73, assuming that point A is located 15° north of west and that the barrel of the gun forms an angle of 25° with the horizontal.

PROBLEM 2.73 A gun is aimed at a point A located 35° east of north. Knowing that the barrel of the gun forms an angle of 40° with the horizontal and that the maximum recoil force is 400 N, determine (*a*) the *x*, *y*, and *z* components of that force, (*b*) the values of the angles θ_x, θ_y, and θ_z defining the direction of the recoil force. (Assume that the *x*, *y*, and *z* axes are directed, respectively, east, up, and south.)

SOLUTION

Recoil force $F = 400 \text{ N}$

$$\therefore \quad F_H = (400 \text{ N})\cos 25° = 362.52 \text{ N}$$



(*a*)

$$F_x = +F_H \cos 15° = +(362.52 \text{ N})\cos 15° = +350.17 \text{ N}$$

$F_x = +350 \text{ N}$ ◀

$$F_y = -F \sin 25° = -(400 \text{ N})\sin 25° = -169.047 \text{ N}$$

$F_y = -169.0 \text{ N}$ ◀

$$F_z = +F_H \sin 15° = +(362.52 \text{ N})\sin 15° = +93.827 \text{ N}$$

$F_z = +93.8 \text{ N}$ ◀

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{+350.17 \text{ N}}{400 \text{ N}}$$

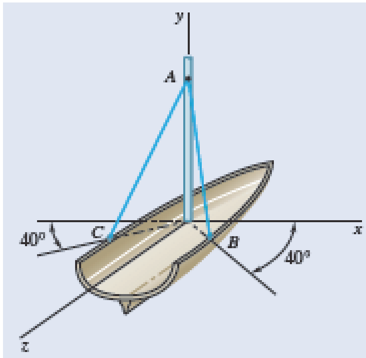
$\theta_x = 28.9°$ ◀

$$\cos\theta_y = \frac{F_y}{F} = \frac{-169.047 \text{ N}}{400 \text{ N}}$$

$\theta_y = 115.0°$ ◀

$$\cos\theta_z = \frac{F_z}{F} = \frac{+93.827 \text{ N}}{400 \text{ N}}$$

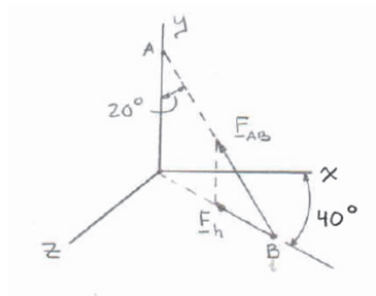
$\theta_z = 76.4°$ ◀



PROBLEM 2.75

The angle between the guy wire *AB* and the mast is 20°. Knowing that the tension in *AB* is 300 lb, determine (*a*) the *x*, *y*, and *z* components of the force exerted on the boat at *B*, (*b*) the angles θ_x, θ_y, and θ_z defining the direction of the force exerted at *B*.

SOLUTION



$$F_h = F_{AB}\sin 20^\circ$$
$$= (300\text{ lb})\sin 20^\circ$$
$$F_h = 102.606\text{ lb}$$

$$F_x = -F_h\cos 40^\circ \qquad F_y = F_{AB}\cos 20^\circ \qquad F_z = -F_h\sin 40^\circ$$
$$F_x = (-102.606\text{ lb})\cos 40^\circ \qquad F_y = (300\text{ lb})\cos 20^\circ \qquad F_z = (-102.606\text{ lb})\sin 40^\circ$$
$$F_x = -78.601\text{ lb} \qquad F_y = 281.91\text{ lb} \qquad F_z = -65.954\text{ lb}$$

(*a*)

$F_x = -78.6\text{ lb}$ ◀

$F_y = 282\text{ lb}$ ◀

$F_z = -66.0\text{ lb}$ ◀

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{-78.601\text{ lb}}{300\text{ lb}}$$

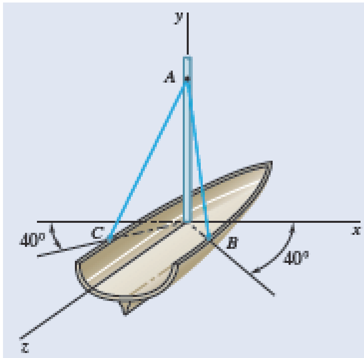
$\theta_x = 105.2^\circ$ ◀

$$\cos\theta_y = \frac{F_y}{F} = \frac{281.91\text{ lb}}{300\text{ lb}}$$

$\theta_y = 20.0^\circ$ ◀

$$\cos\theta_z = \frac{F_z}{F} = \frac{-65.954\text{ lb}}{300\text{ lb}}$$

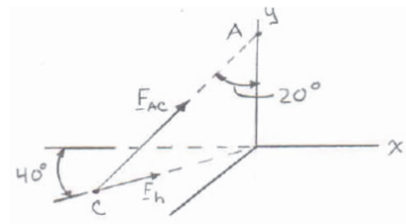
$\theta_z = 102.7^\circ$ ◀



PROBLEM 2.76

The angle between the guy wire *AC* and the mast is 20°. Knowing that the tension in *AC* is 300 lb, determine (*a*) the *x*, *y*, and *z* components of the force exerted on the boat at *C*, (*b*) the angles θ_x, θ_y, and θ_z defining the direction of the force exerted at *C*.

SOLUTION



$$F_h = F_{AC} \sin 20°$$
$$= (300 \text{ lb}) \sin 20°$$
$$F_h = 102.606 \text{ lb}$$

$$F_x = F_h \cos 40° \qquad F_y = F_{AC} \cos 20° \qquad F_z = -F_h \sin 40°$$
$$F_x = (102.606 \text{ lb}) \cos 40° \qquad F_y = (300 \text{ lb}) \cos 20° \qquad F_z = (-102.606 \text{ lb}) \sin 40°$$
$$F_x = 78.601 \text{ lb} \qquad F_y = 281.91 \text{ lb} \qquad F_z = -65.954 \text{ lb}$$

(*a*)

$F_x = 78.6 \text{ lb}$ ◀

$F_y = 282 \text{ lb}$ ◀

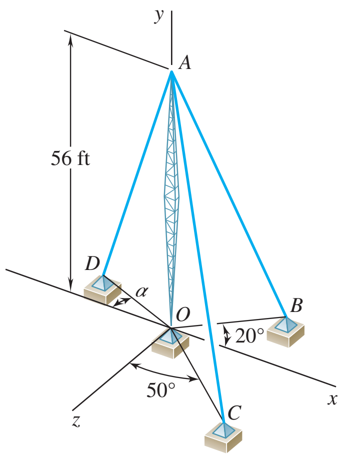
$F_z = -66.0 \text{ lb}$ ◀

(*b*)

$$\cos \theta_x = \frac{F_x}{F} = \frac{78.601 \text{ lb}}{300 \text{ lb}} \qquad \theta_x = 74.8° \blacktriangleleft$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{281.91 \text{ lb}}{300 \text{ lb}} \qquad \theta_y = 20.0° \blacktriangleleft$$

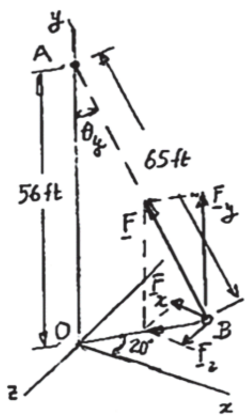
$$\cos \theta_z = \frac{F_z}{F} = \frac{-65.954 \text{ lb}}{300 \text{ lb}} \qquad \theta_z = 102.7° \blacktriangleleft$$



PROBLEM 2.77

Cable *AB* is 65 ft long, and the tension in that cable is 3900 lb. Determine (*a*) the *x*, *y*, and *z* components of the force exerted by the cable on the anchor *B*, (*b*) the angles θ_x, θ_y, and θ_z defining the direction of that force.

SOLUTION



From triangle *AOB*:

$$\cos\theta_y = \frac{56 \text{ ft}}{65 \text{ ft}} = 0.86154$$

$$\theta_y = 30.51°$$

(*a*)

$$F_x = -F\sin\theta_y\cos 20° = -(3900 \text{ lb})\sin 30.51°\cos 20°$$

$$F_x = -1861 \text{ lb} \blacktriangleleft$$

$$F_y = +F\cos\theta_y = (3900 \text{ lb})(0.86154) \qquad F_y = +3360 \text{ lb} \blacktriangleleft$$

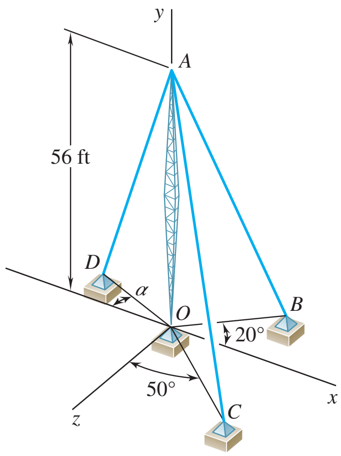
$$F_z = +(3900 \text{ lb})\sin 30.51°\sin 20° \qquad F_z = +677 \text{ lb} \blacktriangleleft$$

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = -\frac{1861 \text{ lb}}{3900 \text{ lb}} = -0.4771 \qquad \theta_x = 118.5° \blacktriangleleft$$

From above: $\theta_y = 30.51°$ $\qquad \theta_y = 30.5° \blacktriangleleft$

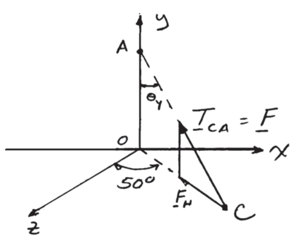
$$\cos\theta_z = \frac{F_z}{F} = +\frac{677 \text{ lb}}{3900 \text{ lb}} = +0.1736 \qquad \theta_z = 80.0° \blacktriangleleft$$



PROBLEM 2.78

Cable *AC* is 70 ft long, and the tension in that cable is 5250 lb. Determine (*a*) the *x*, *y*, and *z* components of the force exerted by the cable on the anchor *C*, (*b*) the angles θ_x, θ_y, and θ_z defining the direction of that force.

SOLUTION



In triangle *AOB*:

$$AC = 70 \text{ ft}$$
$$OA = 56 \text{ ft}$$
$$F = 5250 \text{ lb}$$

$$\cos\theta_y = \frac{56 \text{ ft}}{70 \text{ ft}}$$
$$\theta_y = 36.870°$$
$$F_H = F\sin\theta_y$$
$$= (5250 \text{ lb})\sin 36.870°$$
$$= 3150.0 \text{ lb}$$

(*a*) $F_x = -F_H \sin 50° = -(3150.0 \text{ lb})\sin 50° = -2413.0 \text{ lb}$

$F_x = -2410 \text{ lb}$ ◀

$F_y = +F\cos\theta_y = +(5250 \text{ lb})\cos 36.870° = +4200.0 \text{ lb}$

$F_y = +4200 \text{ lb}$ ◀

$F_z = -F_H \cos 50° = -3150\cos 50° = -2024.8 \text{ lb}$ $\quad F_z = -2025 \text{ lb}$ ◀

(*b*) $\cos\theta_x = \dfrac{F_x}{F} = \dfrac{-2413.0 \text{ lb}}{5250 \text{ lb}}$ $\quad \theta_x = 117.4°$ ◀

From above: $\theta_y = 36.870°$ $\quad \theta_y = 36.9°$ ◀

$\theta_z = \dfrac{F_z}{F} = \dfrac{-2024.8 \text{ lb}}{5250 \text{ lb}}$ $\quad \theta_z = 112.7°$ ◀

PROBLEM 2.79

Determine the magnitude and direction of the force **F** = (260 N)**i** – (320 N)**j** + (800 N)**k**.

SOLUTION

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$F = \sqrt{(260 \text{ N})^2 + (-320 \text{ N})^2 + (800 \text{ N})^2} \qquad F = 900 \text{ N} \blacktriangleleft$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{260 \text{ N}}{900 \text{ N}} \qquad \theta_x = 73.2° \blacktriangleleft$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{-320 \text{ N}}{900 \text{ N}} \qquad \theta_y = 110.8° \blacktriangleleft$$

$$\cos\theta_y = \frac{F_z}{F} = \frac{800 \text{ N}}{900 \text{ N}} \qquad \theta_z = 27.3° \blacktriangleleft$$

PROBLEM 2.80

Determine the magnitude and direction of the force **F** = (700 N)**i** – (820 N)**j** + (960 N)**k.**

SOLUTION

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$F = \sqrt{(700\text{ N})^2 - (820\text{ N})^2 + (960\text{ N})^2}$$

$$F = 1443.61\text{ N}$$

$F = 1444\text{ N}$ ◀

$$\cos\theta_x = \frac{F_x}{F} = \frac{700\text{ N}}{1443.61\text{ N}}$$

$\theta_x = 61.0°$ ◀

$$\cos\theta_y = \frac{F_y}{F} = \frac{-820\text{ N}}{1443.61\text{ N}}$$

$\theta_y = 124.6°$ ◀

$$\cos\theta_y = \frac{F_z}{F} = \frac{960\text{ N}}{1443.61\text{ N}}$$

$\theta_z = 48.3°$ ◀

PROBLEM 2.81

A force **F** of magnitude 250 lb acts at the origin of a coordinate system. Knowing that $\theta_x = 65°$, $\theta_y = 40°$, and $F_z > 0$, determine (*a*) the components of the force, (*b*) the angle θ_z.

SOLUTION

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$\cos^2(65°) + \cos^2(40°) + \cos^2\theta_z = 1$$

$$\cos\theta_z = \pm 0.48432$$

(*b*) Since $F_z > 0$, we choose $\cos\theta_z = 0.48432$ $\quad\therefore\ \theta_z = 61.0°$ ◀

(*a*)

$$F_x = F\cos\theta_x = (250\text{ lb})\cos 65° \qquad F_x = 105.7\text{ lb}$$ ◀

$$F_y = F\cos\theta_y = (250\text{ lb})\cos 40° \qquad F_y = 191.5\text{ lb}$$ ◀

$$F_z = F\cos\theta_z = (250\text{ lb})\cos 61° \qquad F_z = 121.2\text{ lb}$$ ◀

PROBLEM 2.82

A force acts at the origin of a coordinate system in a direction defined by the angles $\theta_x = 70.9°$ and $\theta_y = 144.9°$. Knowing that the z component of the force is s−52.0 lb, determine (*a*) the angle θ_z, (*b*) the other components and the magnitude of the force.

SOLUTION

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$\cos^2 70.9° + \cos^2 144.9° + \cos^2\theta_z° = 1$$

$$\cos\theta_z = \pm 0.47282$$

(*a*) Since $F_z < 0$, we choose $\cos\theta_z = -0.47282$ $\quad\therefore\ \theta_z = 118.2°$ ◀

(*b*)

$$F_z = F\cos\theta_z$$

$$-52.0\ lb = F(-0.47282)$$

$$F = 110.0 \text{ lb}$$ $F = 110.0$ lb ◀

$$F_x = F\cos\theta_x = (110.0 \text{ lb})\cos 70.9°$$ $F_x = 36.0$ lb ◀

$$F_y = F\cos\theta_y = (110.0 \text{ lb})\cos 144.9°$$ $F_y = -90.0$ lb ◀

PROBLEM 2.83

A force **F** of magnitude 210 N acts at the origin of a coordinate system. Knowing that $F_x = 80$ N, $\theta_z = 151.2°$, and $F_y < 0$, determine (*a*) the components F_y and F_z, (*b*) the angles θ_x and θ_y.

SOLUTION

(*a*)

$$F_z = F\cos\theta_z = (210\text{ N})\cos 151.2°$$

$$= -184.024\text{ N} \qquad F_z = -184.0\text{ N} ◀$$

Then:

$$F^2 = F_x^2 + F_y^2 + F_z^2$$

So:

$$(210\text{ N})^2 = (80\text{ N})^2 + (F_y)^2 + (184.024\text{ N})^2$$

Hence:

$$F_y = -\sqrt{(210\text{ N})^2 - (80\text{ N})^2 - (184.024\text{ N})^2}$$

$$= -61.929\text{ N} \qquad F_y = -62.0\text{ lb} ◀$$

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{80\text{ N}}{210\text{ N}} = 0.38095 \qquad \theta_x = 67.6° ◀$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{61.929\text{ N}}{210\text{ N}} = -0.29490$$

$$\theta_y = 107.2° ◀$$

PROBLEM 2.84

A force acts at the origin of a coordinate system in a direction defined by the angles $\theta_y = 120°$ and $\theta_z = 75°$. Knowing that the *x* component of the force is +40 N, determine (*a*) the angle θ_x, (*b*) the magnitude of the force.

SOLUTION

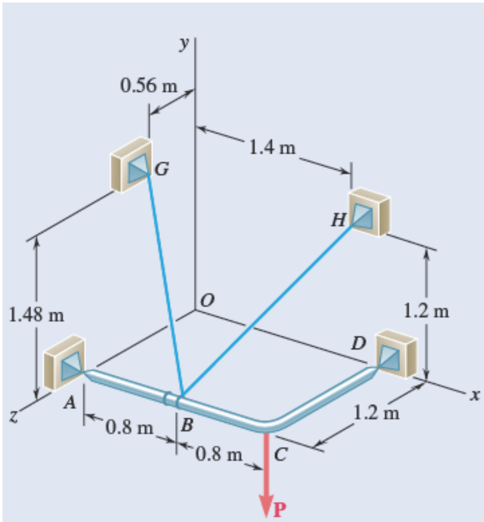
$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$
$$\cos^2\theta_x + \cos^2 120° + \cos^2 75° = 1$$
$$\cos\theta_x = \pm 0.82644$$

(*b*) Since $F_x > 0$, we choose $\cos\theta_x = 0.82644$ $\quad\therefore\ \theta_x = 34.3°$ ◀

(*a*)

$$F_x = F\cos\theta_x$$
$$40\text{ N} = F\cos 34.3°$$

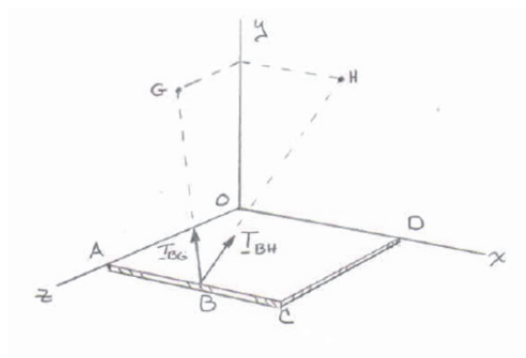
$F = 48.4\text{ N}$ ◀



PROBLEM 2.85

Two cables *BG* and *BH* are attached to frame *ACD* as shown. Knowing that the tension in cable *BG* is 540 N, determine the components of the force exerted by cable *BG* on the frame at *B*.

SOLUTION



$$\overrightarrow{BG} = -(0.8\text{ m})\mathbf{i} + (1.48\text{ m})\mathbf{j} - (0.64\text{ m})\mathbf{k}$$

$$BG = \sqrt{(-0.8\text{ m})^2 + (1.48\text{ m}^2) + (-0.64\text{ m})^2}$$

$$= 1.8\text{ m}$$

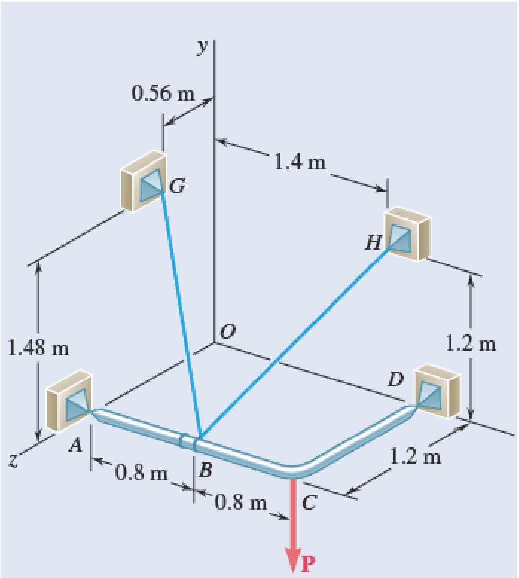
$$\mathbf{T}_{BG} = T_{BG}\boldsymbol{\lambda}_{BG}$$

$$= T_{BG}\frac{\overrightarrow{BG}}{BG}$$

$$= \frac{540\text{ N}}{1.8\text{ m}}[(-0.8\text{ m})\mathbf{i} + (1.48\text{ m})\mathbf{j} + (-0.64\text{ m})\mathbf{k}]$$

$$T_{BG} = (-240\text{ N})\mathbf{i} + (444\text{ N})\mathbf{j} - (192.0\text{ N})\mathbf{k}$$

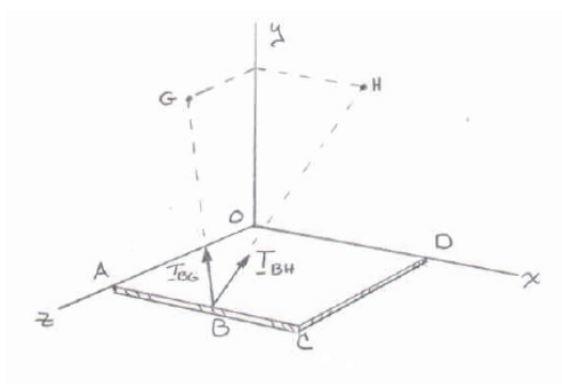
$$F_x = -240\text{ N},\quad F_y = +444\text{ N},\quad F_z = +192.0\text{ N} \blacktriangleleft$$



PROBLEM 2.86

Two cables *BG* and *BH* are attached to frame *ACD* as shown. Knowing that the tension in cable *BH* is 750 N, determine the components of the force exerted by cable *BH* on the frame at *B*.

SOLUTION



$$\overrightarrow{BH} = (0.6 \text{ m})\mathbf{i} + (1.2 \text{ m})\mathbf{j} - (1.2 \text{ m})\mathbf{k}$$

$$BH = \sqrt{(0.6 \text{ m})^2 + (1.2 \text{ m})^2 - (1.2 \text{ m})^2}$$

$$= 1.8 \text{ m}$$

$$\mathbf{T}_{BH} = T_{BH}\boldsymbol{\lambda}_{BH}$$

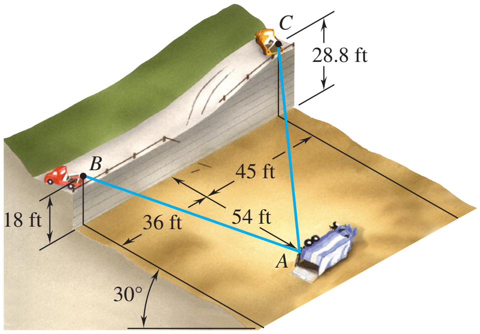
$$= T_{BH}\frac{\overrightarrow{BH}}{BH}$$

$$= \frac{750 \text{ N}}{3 \text{ m}}[\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}] \text{ (m)}$$

$$T_{BH} = (250 \text{ N})\mathbf{i} + (500 \text{ N})\mathbf{j} - (500 \text{ N})\mathbf{k}$$

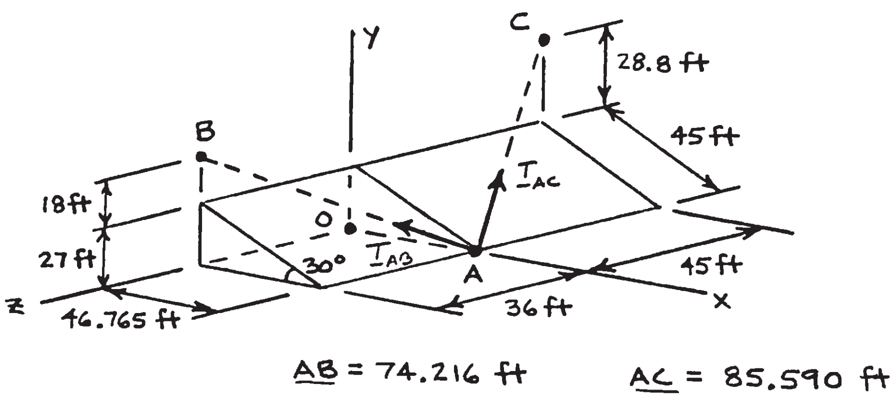
$F_x = +250$ N, $F_y = +500$ N, $F_z = -500$ N ◀

PROBLEM 2.87



In order to move a wrecked truck, two cables are attached at A and pulled by winches B and C as shown. Knowing that the tension in cable AB is 2 kips, determine the components of the force exerted at A by the cable.

SOLUTION



Cable AB:

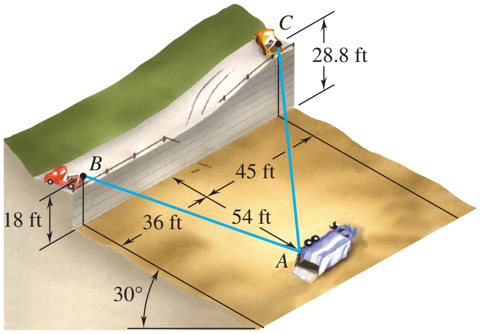
$$\boldsymbol{\lambda}_{AB} = \frac{\overrightarrow{AB}}{AB} = \frac{(-46.765\text{ ft})\mathbf{i} + (45\text{ ft})\mathbf{j} + (36\text{ ft})\mathbf{k}}{74.216\text{ ft}}$$

$$\mathbf{T}_{AB} = T_{AB}\,\boldsymbol{\lambda}_{AB} = \frac{-46.765\mathbf{i} + 45\mathbf{j} + 36\mathbf{k}}{74.216}$$

$$(T_{AB})_x = -1.260\text{ kips} \blacktriangleleft$$

$$(T_{AB})_y = +1.213\text{ kips} \blacktriangleleft$$

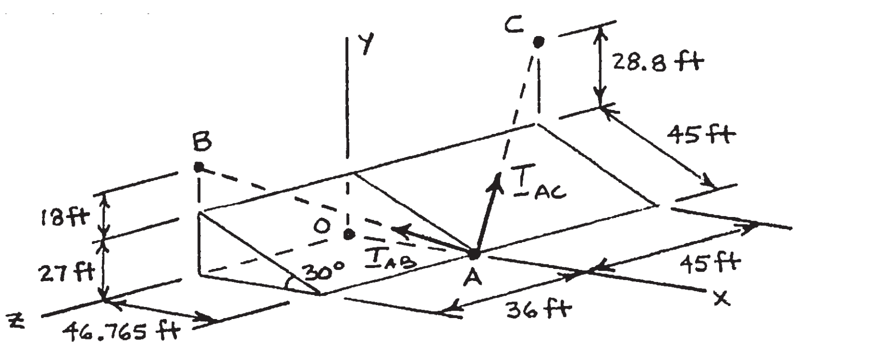
$$(T_{AB})_z = +0.970\text{ kips} \blacktriangleleft$$



PROBLEM 2.88

In order to move a wrecked truck, two cables are attached at *A* and pulled by winches *B* and *C* as shown. Knowing that the tension in cable *AC* is 1.5 kips, determine the components of the force exerted at *A* by the cable.

SOLUTION



AB = 74.216 ft AC = 85.590 ft

Cable *AB*:

$$\boldsymbol{\lambda}_{AC} = \frac{\overrightarrow{AC}}{AC} = \frac{(-46.765 \text{ ft})\mathbf{i} + (55.8 \text{ ft})\mathbf{j} + (-45 \text{ ft})\mathbf{k}}{85.590 \text{ ft}}$$

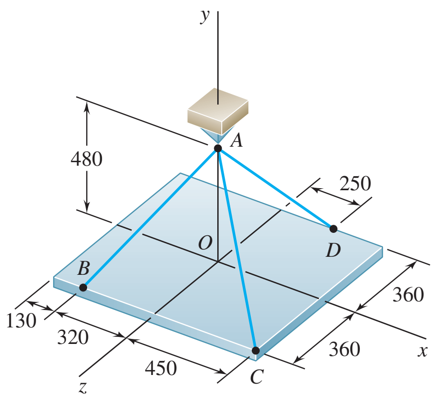
$$\mathbf{T}_{AC} = T_{AC}\,\boldsymbol{\lambda}_{AC} = (1.5 \text{ kips})\frac{-46.765\mathbf{i} + 55.8\mathbf{j} - 45\mathbf{k}}{85.590}$$

$(T_{AC})_x = -0.820$ kips ◀

$(T_{AC})_y = +0.978$ kips ◀

$(T_{AC})_z = -0.789$ kips ◀

PROBLEM 2.89



Dimensions in mm

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable AB is 408 N, determine the components of the force exerted on the plate at B.

SOLUTION

We have:

$$\overrightarrow{BA} = +(320 \text{ mm})\mathbf{i} + (480 \text{ mm})\mathbf{j} - (360 \text{ mm})\mathbf{k} \quad BA = 680 \text{ mm}$$

Thus:

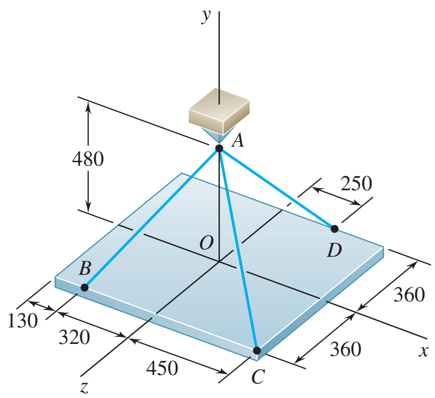
$$\mathbf{F}_B = T_{BA}\boldsymbol{\lambda}_{BA} = T_{BA}\frac{\overrightarrow{BA}}{BA} = T_{BA}\left(\frac{8}{17}\mathbf{i} + \frac{12}{17}\mathbf{j} - \frac{9}{17}\mathbf{k}\right)$$

$$\left(\frac{8}{17}T_{BA}\right)\mathbf{i} + \left(\frac{12}{17}T_{BA}\right)\mathbf{j} - \left(\frac{9}{17}T_{BA}\right)\mathbf{k} = 0$$

Setting $T_{BA} = 408 \text{ N}$ yields,

$$F_x = +192.0 \text{ N}, \quad F_y = +288 \text{ N}, \quad F_z = -216 \text{ N} ◀$$

PROBLEM 2.90



Dimensions in mm

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable AD is 429 N, determine the components of the force exerted on the plate at D.

SOLUTION

We have:

$$\overrightarrow{DA} = -(250 \text{ mm})\mathbf{i} + (480 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k} \quad DA = 650 \text{ mm}$$

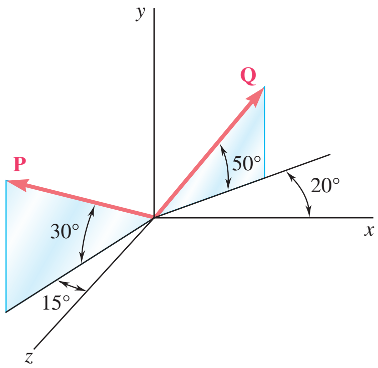
Thus:

$$\mathrm{F}_D = T_{DA}\boldsymbol{\lambda}_{DA} = T_{DA}\frac{\overrightarrow{DA}}{DA} = T_{DA}\left(-\frac{5}{13}\mathbf{i} + \frac{48}{65}\mathbf{j} + \frac{36}{65}\mathbf{k}\right)$$

$$-\left(\frac{5}{13}T_{DA}\right)\mathbf{i} + \left(\frac{48}{65}T_{DA}\right)\mathbf{j} + \left(\frac{36}{65}T_{DA}\right)\mathbf{k} = 0$$

Setting $T_{DA} = 429$ N yields,

$$F_x = -165.0 \text{ N}, \quad F_y = +317 \text{ N}, \quad F_z = +238 \text{ N} \blacktriangleleft$$



PROBLEM 2.91

Find the magnitude and direction of the resultant of the two forces shown knowing that $P = 300$ N and $Q = 400$ N.

SOLUTION

$$\mathbf{P} = (300\text{ N})[-\cos 30^\circ \sin 15^\circ \mathbf{i} + \sin 30^\circ \mathbf{j} + \cos 30^\circ \cos 15^\circ \mathbf{k}]$$

$$= -(67.243\text{ N})\mathbf{i} + (150\text{ N})\mathbf{j} + (250.95\text{ N})\mathbf{k}$$

$$\mathbf{Q} = (400\text{ N})[\cos 50^\circ \cos 20^\circ \mathbf{i} + \sin 50^\circ \mathbf{j} - \cos 50^\circ \sin 20^\circ \mathbf{k}]$$

$$= (400\text{ N})[0.60402\mathbf{i} + 0.76604\mathbf{j} - 0.21985]$$

$$= (241.61\text{ N})\mathbf{i} + (306.42\text{ N})\mathbf{j} - (87.939\text{ N})\mathbf{k}$$

$$\mathbf{R} = \mathbf{P} + \mathbf{Q}$$

$$= (174.367\text{ N})\mathbf{i} + (456.42\text{ N})\mathbf{j} + (163.011\text{ N})\mathbf{k}$$

$$R = \sqrt{(174.367\text{ N})^2 + (456.42\text{ N})^2 + (163.011\text{ N})^2}$$

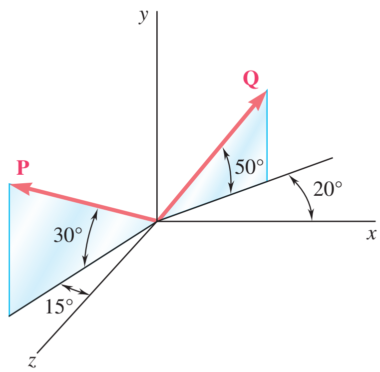
$$= 515.07\text{ N} \qquad R = 515\text{ N} \blacktriangleleft$$

$$\cos\theta_x = \frac{R_x}{R} = \frac{174.367\text{ N}}{515.07\text{ N}} = 0.33853 \qquad \theta_x = 70.2^\circ \blacktriangleleft$$

$$\cos\theta_y = \frac{R_y}{R} = \frac{456.42\text{ N}}{515.07\text{ N}} = 0.88613 \qquad \theta_y = 27.6^\circ \blacktriangleleft$$

$$\cos\theta_z = \frac{R_z}{R} = \frac{163.011\text{ N}}{515.07\text{ N}} = 0.31648 \qquad \theta_z = 71.5^\circ \blacktriangleleft$$

PROBLEM 2.92



Find the magnitude and direction of the resultant of the two forces shown knowing that $P = 400$ N and $Q = 300$ N.

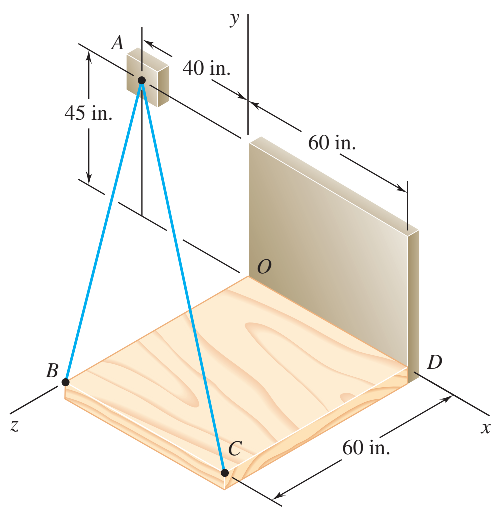
SOLUTION

$$\mathbf{P} = (400\text{ N})[-\cos 30^\circ \sin 15^\circ \mathbf{i} + \sin 30^\circ \mathbf{j} + \cos 30^\circ \cos 15^\circ \mathbf{k}]$$
$$= -(89.678\text{ N})\mathbf{i} + (200\text{ N})\mathbf{j} + (334.61\text{ N})\mathbf{k}$$
$$\mathbf{Q} = (300\text{ N})[\cos 50^\circ \cos 20^\circ \mathbf{i} + \sin 50^\circ \mathbf{j} - \cos 50^\circ \sin 20^\circ \mathbf{k}]$$
$$= (181.21\text{ N})\mathbf{i} + (229.81\text{ N})\mathbf{j} - (65.954\text{ N})\mathbf{k}$$
$$\mathbf{R} = \mathbf{P} + \mathbf{Q}$$
$$= (91.532\text{ N})\mathbf{i} + (429.81\text{ N})\mathbf{j} + (268.66\text{ N})\mathbf{k}$$
$$R = \sqrt{(91.532\text{ N})^2 + (429.81\text{ N})^2 + (268.66\text{ N})^2}$$
$$= 515.07\text{ N} \qquad R = 515\text{ N} \blacktriangleleft$$

$$\cos\theta_x = \frac{R_x}{R} = \frac{91.532\text{ N}}{515.07\text{ N}} = 0.177708 \qquad \theta_x = 79.8^\circ \blacktriangleleft$$

$$\cos\theta_y = \frac{R_y}{R} = \frac{429.81\text{ N}}{515.07\text{ N}} = 0.83447 \qquad \theta_y = 33.4^\circ \blacktriangleleft$$

$$\cos\theta_z = \frac{R_z}{R} = \frac{268.66\text{ N}}{515.07\text{ N}} = 0.52160 \qquad \theta_z = 58.6^\circ \blacktriangleleft$$



PROBLEM 2.93

Knowing that the tension is 425 lb in cable *AB* and 510 lb in cable *AC*, determine the magnitude and direction of the resultant of the forces exerted at *A* by the two cables.

SOLUTION

$$\overrightarrow{AB} = (40\text{ in.})\mathbf{i} - (45\text{ in.})\mathbf{j} + (60\text{ in.})\mathbf{k}$$

$$AB = \sqrt{(40\text{ in.})^2 + (45\text{ in.})^2 + (60\text{ in.})^2} = 85\text{ in.}$$

$$\overrightarrow{AC} = (100\text{ in.})\mathbf{i} - (45\text{ in.})\mathbf{j} + (60\text{ in.})\mathbf{k}$$

$$AC = \sqrt{(100\text{ in.})^2 + (45\text{ in.})^2 + (60\text{ in.})^2} = 125\text{ in.}$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = (425\text{ lb})\left[\frac{(40\text{ in.})\mathbf{i} - (45\text{ in.})\mathbf{j} + (60\text{ in.})\mathbf{k}}{85\text{ in.}}\right]$$

$$\mathbf{T}_{AB} = (200\text{ lb})\mathbf{i} - (225\text{ lb})\mathbf{j} + (300\text{ lb})\mathbf{k}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = (510\text{ lb})\left[\frac{(100\text{ in.})\mathbf{i} - (45\text{ in.})\mathbf{j} + (60\text{ in.})\mathbf{k}}{125\text{ in.}}\right]$$

$$\mathbf{T}_{AC} = (408\text{ lb})\mathbf{i} - (183.6\text{ lb})\mathbf{j} + (244.8\text{ lb})\mathbf{k}$$

$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} = (608)\mathbf{i} - (408.6\text{ lb})\mathbf{j} + (544.8\text{ lb})\mathbf{k}$$

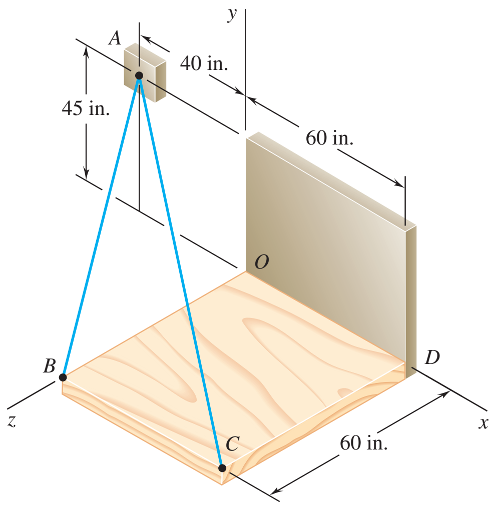
Then: $R = 912.92\text{ lb}$ $\quad R = 913\text{ lb}$ ◀

and

$$\cos\theta_x = \frac{608\text{ lb}}{912.92\text{ lb}} = 0.66599 \qquad \theta_x = 48.2° ◀$$

$$\cos\theta_y = \frac{408.6\text{ lb}}{912.92\text{ lb}} = -0.44757 \qquad \theta_y = 116.6° ◀$$

$$\cos\theta_z = \frac{544.8\text{ lb}}{912.92\text{ lb}} = 0.59677 \qquad \theta_z = 53.4° ◀$$



PROBLEM 2.94

Knowing that the tension is 510 lb in cable AB and 425 lb in cable AC, determine the magnitude and direction of the resultant of the forces exerted at A by the two cables.

SOLUTION

$$\overrightarrow{AB} = (40\text{ in.})\mathbf{i} - (45\text{ in.})\mathbf{j} + (60\text{ in.})\mathbf{k}$$

$$AB = \sqrt{(40\text{ in.})^2 + (45\text{ in.})^2 + (60\text{ in.})^2} = 85\text{ in.}$$

$$\overrightarrow{AC} = (100\text{ in.})\mathbf{i} - (45\text{ in.})\mathbf{j} + (60\text{ in.})\mathbf{k}$$

$$AC = \sqrt{(100\text{ in.})^2 + (45\text{ in.})^2 + (60\text{ in.})^2} = 125\text{ in.}$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = (510\text{ lb})\left[\frac{(40\text{ in.})\mathbf{i} - (45\text{ in.})\mathbf{j} + (60\text{ in.})\mathbf{k}}{85\text{ in.}}\right]$$

$$\mathbf{T}_{AB} = (240\text{ lb})\mathbf{i} - (270\text{ lb})\mathbf{j} + (360\text{ lb})\mathbf{k}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = (425\text{ lb})\left[\frac{(100\text{ in.})\mathbf{i} - (45\text{ in.})\mathbf{j} + (60\text{ in.})\mathbf{k}}{125\text{ in.}}\right]$$

$$\mathbf{T}_{AC} = (340\text{ lb})\mathbf{i} - (153\text{ lb})\mathbf{j} + (204\text{ lb})\mathbf{k}$$

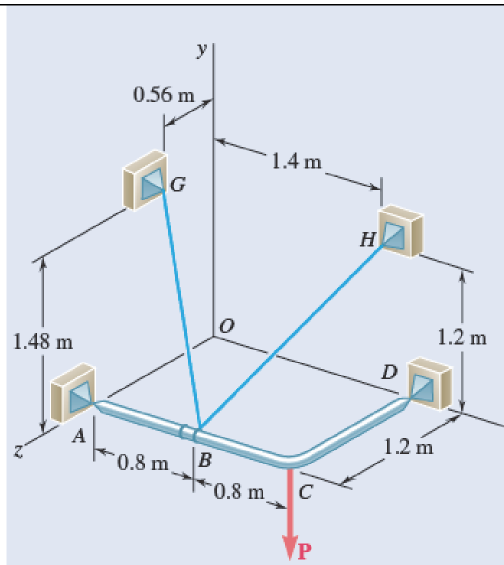
$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} = (580\text{ lb})\mathbf{i} - (423\text{ lb})\mathbf{j} + (564\text{ lb})\mathbf{k}$$

Then: $R = 912.92\text{ lb}$ $\qquad R = 913\text{ lb}$ ◀

and $\cos\theta_x = \dfrac{580\text{ lb}}{912.92\text{ lb}} = 0.63532$ $\qquad \theta_x = 50.6°$ ◀

$\cos\theta_y = \dfrac{-423\text{ lb}}{912.92\text{ lb}} = -0.46335$ $\qquad \theta_y = 117.6°$ ◀

$\cos\theta_z = \dfrac{564\text{ lb}}{912.92\text{ lb}} = 0.61780$ $\qquad \theta_z = 51.8°$ ◀

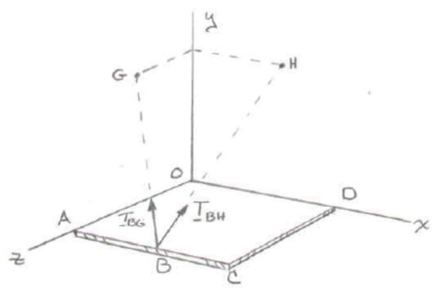


PROBLEM 2.95

For the frame of Prob. 2.85, determine the magnitude and direction of the resultant of the forces exerted by the cables at *B* knowing that the tension is 540 N in cable *BG* and 750 N in cable *BH*.

PROBLEM 2.85 Two cables BG and BH are attached to frame ACD as shown. Knowing that the tension in cable BG is 540 N, determine the components of the force exerted by cable BG on the frame at B.

SOLUTION



$$\overrightarrow{BG} = -(0.8\text{ m})\mathbf{i} + (1.48\text{ m})\mathbf{j} - (0.64\text{ m})\mathbf{k}$$

$$BG = \sqrt{(-0.8\text{ m})^2 + (1.48\text{ m}^2) + (-0.64\text{ m})^2} = 1.8\text{ m}$$

$$\overrightarrow{BH} = (0.6\text{ m})\mathbf{i} + (1.2\text{ m})\mathbf{j} - (1.2\text{ m})\mathbf{k}$$

$$BH = \sqrt{(0.6\text{ m})^2 + (1.2\text{ m})^2 - (1.2\text{ m})^2} = 1.8\text{ m}$$

$$\mathbf{T}_{BG} = T_{BG}\boldsymbol{\lambda}_{BG}$$

$$= T_{BG}\frac{\overrightarrow{BG}}{BG} = \frac{540\text{ N}}{1.8\text{ m}}[(-0.8\text{ m})\mathbf{i} + (1.48\text{ m})\mathbf{j} + (-0.64\text{ m})\mathbf{k}]$$

$$= (-240\text{ N})\mathbf{i} + (444\text{ N})\mathbf{j} - (192.0\text{ N})\mathbf{k}$$

$$\mathbf{T}_{BH} = T_{BH}\boldsymbol{\lambda}_{BH}$$

$$= T_{BH}\frac{\overrightarrow{BH}}{BH} = \frac{750\text{ N}}{3\text{ m}}[\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}]\text{ (m)}$$

$$\mathbf{T}_{BH} = (250\text{ N})\mathbf{i} + (500\text{ N})\mathbf{j} - (500\text{ N})\mathbf{k}$$

$$\mathbf{R} = \mathbf{T}_{BG} + \mathbf{T}_{BH} = (10\text{ N})\mathbf{i} + (944\text{ N})\mathbf{j} - (692\text{ N})\mathbf{k}$$

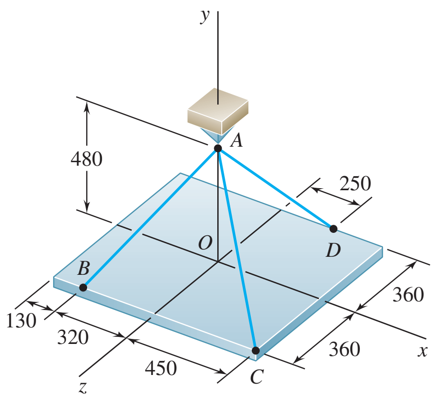
Then: $R = \sqrt{\left(10^2 + 944^2 + (-692)^2\right)} = 1170.51\text{ N}$ $\qquad R = 1171\text{ N}$ ◀

and $\cos\theta_x = \dfrac{10\text{ N}}{1170.51\text{ N}}$ $\qquad \theta_x = 89.5°$ ◀

$\cos\theta_y = \dfrac{944\text{ N}}{1170.51\text{ N}}$ $\qquad \theta_y = 36.2°$ ◀

$\cos\theta_z = \dfrac{-692\text{ N}}{1170.51\text{ N}}$ $\qquad \theta_z = 126.2°$ ◀

PROBLEM 2.96



For the plate of Prob. 2.89, determine the tensions in cables *AB* and *AD* knowing that the tension in cable *AC* is 54 N and that the resultant of the forces exerted by the three cables at *A* must be vertical.

SOLUTION

We have:

$$\overrightarrow{AB} = -(320\text{ mm})\mathbf{i} - (480\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k} \quad AB = 680\text{ mm}$$

$$\overrightarrow{AC} = (450\text{ mm})\mathbf{i} - (480\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k} \quad AC = 750\text{ mm}$$

$$\overrightarrow{AD} = (250\text{ mm})\mathbf{i} - (480\text{ mm})\mathbf{j} - (360\text{ mm})\mathbf{k} \quad AD = 650\text{ mm}$$

Thus:

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{T_{AB}}{680}(-320\mathbf{i} - 480\mathbf{j} + 360\mathbf{k})$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{54}{750}(450\mathbf{i} - 480\mathbf{j} + 360\mathbf{k})$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{650}(250\mathbf{i} - 480\mathbf{j} - 360\mathbf{k})$$

Substituting into the Eq. $\mathbf{R} = \Sigma\mathbf{F}$ and factoring $\mathbf{i}, \mathbf{j}, \mathbf{k}$:

$$\mathbf{R} = \left(-\frac{320}{680}T_{AB} + 32.40 + \frac{250}{650}T_{AD}\right)\mathbf{i}$$

$$+\left(-\frac{480}{680}T_{AB} - 34.560 - \frac{480}{650}T_{AD}\right)\mathbf{j}$$

$$+\left(\frac{360}{680}T_{AB} + 25.920 - \frac{360}{650}T_{AD}\right)\mathbf{k}$$

PROBLEM 2.96 (Continued)

Since **R** is vertical, the coefficients of **i** and **k** are zero:

$$\mathbf{i}: \quad -\frac{320}{680}T_{AB} + 32.40 + \frac{250}{650}T_{AD} = 0 \qquad (1)$$

$$\mathbf{k}: \quad \frac{360}{680}T_{AB} + 25.920 - \frac{360}{650}T_{AD} = 0 \qquad (2)$$

Multiply (1) by 3.6 and (2) by 2.5 then add:

$$-\frac{252}{680}T_{AB} + 181.440 = 0$$

$$T_{AB} = 489.60 \text{ N}$$

$$T_{AB} = 490 \text{ N} \blacktriangleleft$$

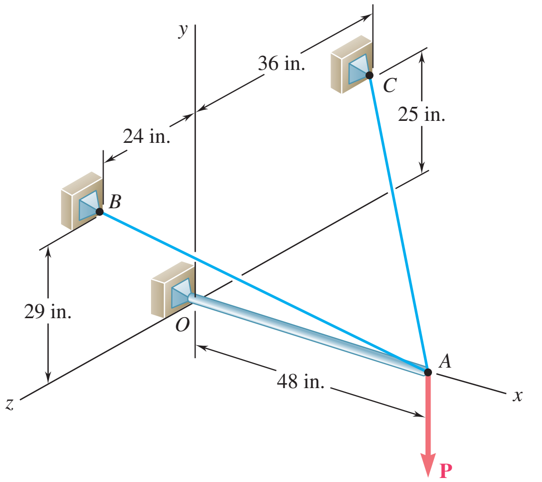
Substitute into (2) and solve for T_{AD}:

$$\frac{360}{680}(489.60 \text{ N}) + 25.920 - \frac{360}{650}T_{AD} = 0$$

$$T_{AD} = 514.80 \text{ N}$$

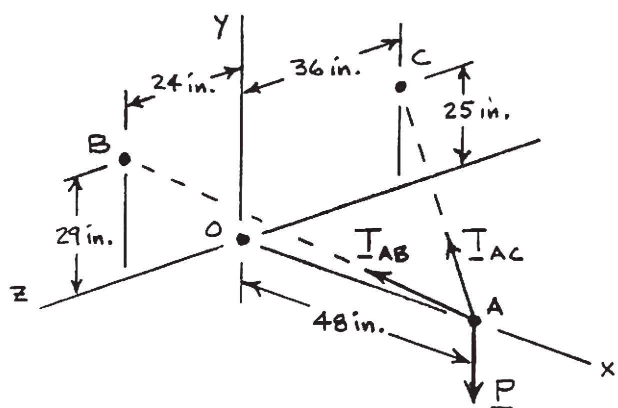
$$T_{AD} = 515 \text{ N} \blacktriangleleft$$

PROBLEM 2.97



The boom *OA* carries a load **P** and is supported by two cables as shown. Knowing that the tension in cable *AB* is 183 lb and that the resultant of the load **P** and of the forces exerted at *A* by the two cables must be directed along *OA*, determine the tension in cable *AC*.

SOLUTION



Cable *AB*: $T_{AB} = 183 \text{ lb}$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = (183 \text{ lb})\frac{(-48 \text{ in.})\mathbf{i} + (29 \text{ in.})\mathbf{j} + (24 \text{ in.})\mathbf{k}}{61 \text{ in.}}$$

$$\mathbf{T}_{AB} = -(144 \text{ lb})\mathbf{i} + (87 \text{ lb})\mathbf{j} + (72 \text{ lb})\mathbf{k}$$

Cable *AC*:

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = T_{AC}\frac{(-48 \text{ in.})\mathbf{i} + (25 \text{ in.})\mathbf{j} + (-36 \text{ in.})\mathbf{k}}{65 \text{ in.}}$$

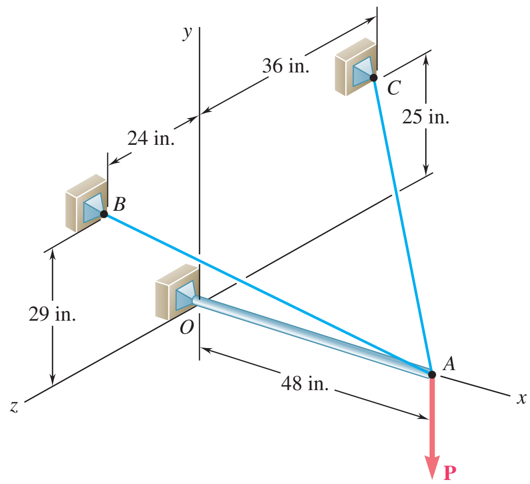
$$\mathbf{T}_{AC} = -\frac{48}{65}T_{AC}\mathbf{i} + \frac{25}{65}T_{AC}\mathbf{j} - \frac{36}{65}T_{AC}\mathbf{k}$$

Load *P*: $\mathbf{P} = P\,\mathbf{j}$

For resultant to be directed along *OA*, i.e., *x*-axis

$$R_z = 0: \quad \Sigma F_z = (72 \text{ lb}) - \frac{36}{65}T'_{AC} = 0$$

$T_{AC} = 130.0 \text{ lb}$ ◀



PROBLEM 2.98

For the boom and loading of Problem. 2.97, determine the magnitude of the load **P**.

PROBLEM 2.97 The boom *OA* carries a load **P** and is supported by two cables as shown. Knowing that the tension in cable *AB* is 183 lb and that the resultant of the load **P** and of the forces exerted at *A* by the two cables must be directed along *OA*, determine the tension in cable *AC*.

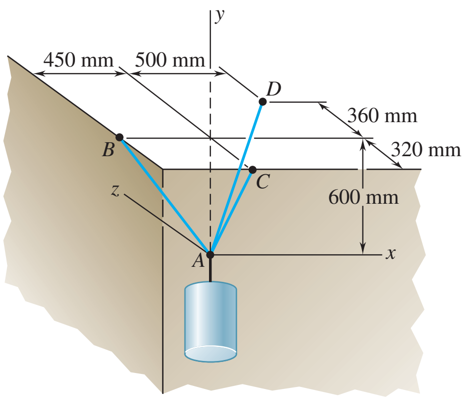
SOLUTION

See Problem 2.97. Since resultant must be directed along *OA*, i.e., the *x*-axis, we write

$$R_y = 0: \quad \Sigma F_y = (87 \text{ lb}) + \frac{25}{65} T_{AC} - P = 0$$

$T_{AC} = 130.0$ lb from Problem 2.97.

Then $$(87 \text{ lb}) + \frac{25}{65}(130.0 \text{ lb}) - P = 0$$ $P = 137.0$ lb ◀

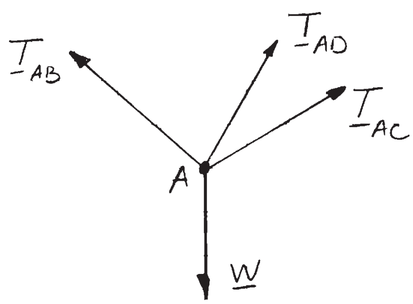


PROBLEM 2.99

A container is supported by three cables that are attached to a ceiling as shown. Determine the weight W of the container, knowing that the tension in cable AB is 6 kN.

SOLUTION

Free-Body Diagram at A:



The forces applied at A are: $\mathbf{T}_{AB}, \mathbf{T}_{AC}, \mathbf{T}_{AD}$, and $\mathbf{W}$

where $\mathbf{W} = W\,\mathbf{j}$. To express the other forces in terms of the unit vectors **i**, **j**, **k**, we write

$$\overrightarrow{AB} = -(450\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j} \qquad AB = 750\text{ mm}$$

$$\overrightarrow{AC} = +(600\text{ mm})\mathbf{j} - (320\text{ mm})\mathbf{k} \qquad AC = 680\text{ mm}$$

$$\overrightarrow{AD} = +(500\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k} \qquad AD = 860\text{ mm}$$

and

$$\mathbf{T}_{AB} = \boldsymbol{\lambda}_{AB}T_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = T_{AB}\frac{(-450\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j}}{750\text{ mm}}$$

$$= \left(-\frac{45}{75}\mathbf{i} + \frac{60}{75}\mathbf{j}\right)T_{AB}$$

$$\mathbf{T}_{AC} = \boldsymbol{\lambda}_{AC}T_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = T_{AC}\frac{(600\text{ mm})\mathbf{i} - (320\text{ mm})\mathbf{j}}{680\text{ mm}}$$

$$= \left(\frac{60}{68}\mathbf{j} - \frac{32}{68}\mathbf{k}\right)T_{AC}$$

$$\mathbf{T}_{AD} = \boldsymbol{\lambda}_{AD}T_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = T_{AD}\frac{(500\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k}}{860\text{ mm}}$$

$$= \left(\frac{50}{86}\mathbf{i} + \frac{60}{86}\mathbf{j} + \frac{36}{86}\mathbf{k}\right)T_{AD}$$

PROBLEM 2.99 (Continued)

Equilibrium condition: $\Sigma F = 0: \quad \therefore \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{W} = 0$

Substituting the expressions obtained for $\mathbf{T}_{AB}$, $\mathbf{T}_{AC}$, and $\mathbf{T}_{AD}$; factoring **i**, **j**, and **k**; and equating each of the coefficients to zero gives the following equations:

From **i**: $$-\frac{45}{75}T_{AB} + \frac{50}{86}T_{AD} = 0 \qquad (1)$$

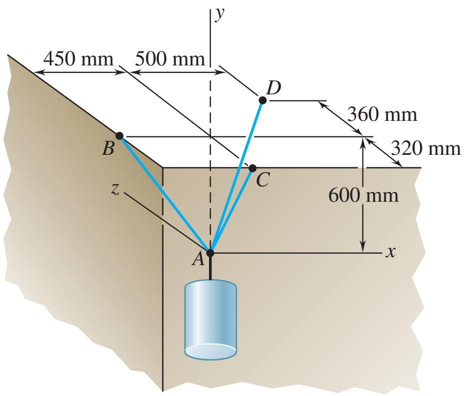
From **j**: $$\frac{60}{75}T_{AB} + \frac{60}{68}T_{AC} + \frac{60}{86}T_{AD} - W = 0 \qquad (2)$$

From **k**: $$-\frac{32}{68}T_{AC} + \frac{36}{86}T_{AD} = 0 \qquad (3)$$

Setting $T_{AB} = 6 \text{ kN}$ in (1) and (2), and solving the resulting set of equations gives

$$T_{AC} = 6.1920 \text{ kN}$$
$$T_{AC} = 5.5080 \text{ kN}$$

$W = 13.98 \text{ kN}$ ◀



PROBLEM 2.100

A container is supported by three cables that are attached to a ceiling as shown. Determine the weight W of the container, knowing that the tension in cable AD is 4.3 kN.

SOLUTION

See Problem 2.99 for the figure and analysis leading to the following set of linear algebraic equations:

$$-\frac{45}{75}T_{AB} + \frac{50}{86}T_{AD} = 0 \qquad (1)$$

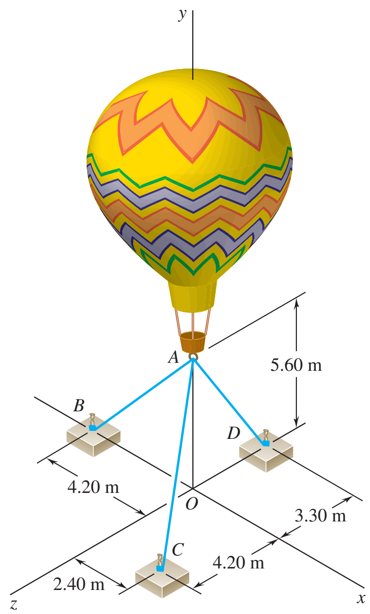
$$\frac{60}{75}T_{AB} + \frac{60}{68}T_{AC} + \frac{60}{86}T_{AD} - W = 0 \qquad (2)$$

$$-\frac{32}{68}T_{AC} + \frac{36}{86}T_{AD} = 0 \qquad (3)$$

Setting $T_{AD} = 4.3$ kN into the above equations gives

$$T_{AB} = 4.1667 \text{ kN}$$
$$T_{AC} = 3.8250 \text{ kN}$$

$W = 9.71$ kN ◀

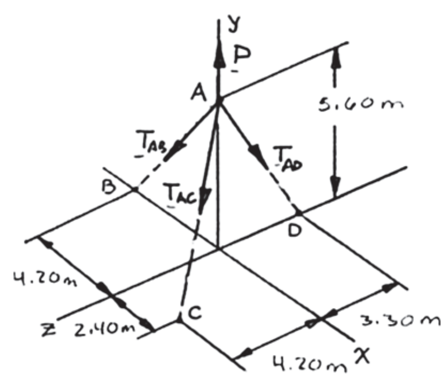


PROBLEM 2.101

Three cables are used to tether a balloon as shown. Determine the vertical force **P** exerted by the balloon at *A* knowing that the tension in cable *AD* is 481 N.

SOLUTION

FREE-BODY DIAGRAM AT *A*



The forces applied at *A* are: $\mathbf{T}_{AB}, \mathbf{T}_{AC}, \mathbf{T}_{AD}$, and $\mathbf{P}$

where $\mathbf{P} = P\mathbf{j}$. To express the other forces in terms of the unit vectors **i**, **j**, **k**, we write

$$\overrightarrow{AB} = -(4.20 \text{ m})\mathbf{i} - (5.60 \text{ m})\mathbf{j} \qquad AB = 7.00 \text{ m}$$

$$\overrightarrow{AC} = (2.40 \text{ m})\mathbf{i} - (5.60 \text{ m})\mathbf{j} + (4.20 \text{ m})\mathbf{k} \qquad AC = 7.40 \text{ m}$$

$$\overrightarrow{AD} = -(5.60 \text{ m})\mathbf{j} - (3.30 \text{ m})\mathbf{k} \qquad AD = 6.50 \text{ m}$$

and

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = (-0.6\mathbf{i} - 0.8\mathbf{j})T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = (0.32432\mathbf{i} - 0.75676\mathbf{j} + 0.56757\mathbf{k})T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = (-0.86154\mathbf{j} - 0.50769\mathbf{k})T_{AD}$$

PROBLEM 2.101 (Continued)

Equilibrium condition: $\Sigma F = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$

Substituting the expressions obtained for $\mathbf{T}_{AB}$, $\mathbf{T}_{AC}$, and $\mathbf{T}_{AD}$ and factoring **i**, **j**, and **k**:

$$(-0.6T_{AB} + 0.32432T_{AC})\mathbf{i} + (-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P)\mathbf{j}$$
$$+(0.56757T_{AC} - 0.50769T_{AD})\mathbf{k} = 0$$

Equating to zero the coefficients of **i**, **j**, **k**:

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \quad (1)$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \quad (2)$$

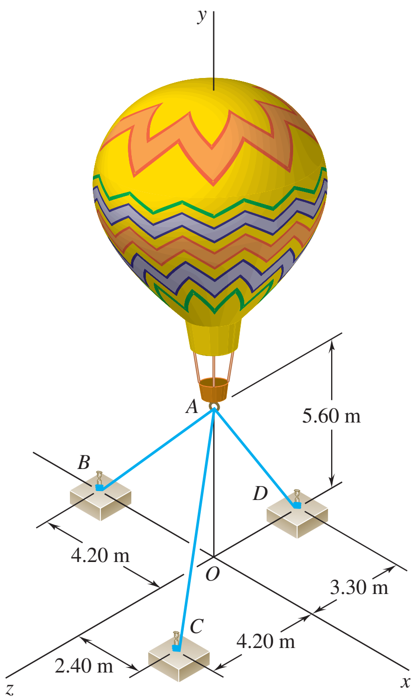
$$0.56757T_{AC} - 0.50769T_{AD} = 0 \quad (3)$$

Setting $T_{AD} = 481$ N in (2) and (3), and solving the resulting set of equations gives

$$T_{AC} = 430.26 \text{ N}$$
$$T_{AD} = 232.57 \text{ N}$$

P = 926 N ↑ ◀

PROBLEM 2.102



Three cables are used to tether a balloon as shown. Knowing that the balloon exerts an 800-N vertical force at *A*, determine the tension in each cable.

SOLUTION

See Problem 2.101 for the figure and analysis leading to the linear algebraic Equations (1), (2), and (3).

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \qquad (1)$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \qquad (2)$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \qquad (3)$$

From Eq. (1): $T_{AB} = 0.54053T_{AC}$

From Eq. (3): $T_{AD} = 1.11795T_{AC}$

Substituting for T_{AB} and T_{AD} in terms of T_{AC} into Eq. (2) gives

$$-0.8(0.54053T_{AC}) - 0.75676T_{AC} - 0.86154(1.11795T_{AC}) + P = 0$$

$$2.1523T_{AC} = P; \quad P = 800 \text{ N}$$

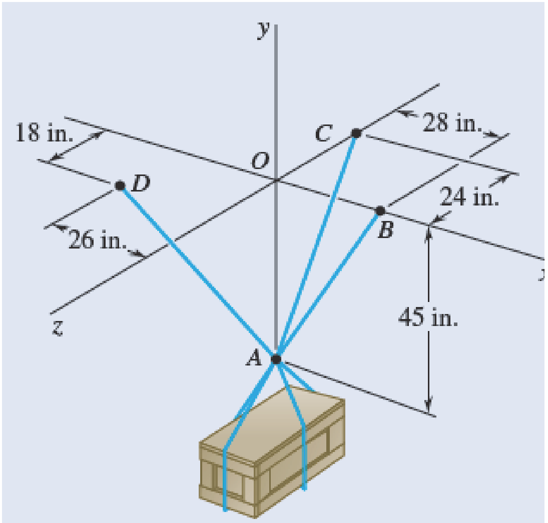
$$T_{AC} = \frac{800 \text{ N}}{2.1523} = 371.69 \text{ N}$$

Substituting into expressions for T_{AB} and T_{AD} gives

$$T_{AB} = 0.54053(371.69 \text{ N})$$

$$T_{AD} = 1.11795(371.69 \text{ N})$$

$$T_{AB} = 201 \text{ N}, \quad T_{AC} = 372 \text{ N}, \quad T_{AD} = 416 \text{ N} \blacktriangleleft$$



PROBLEM 2.103

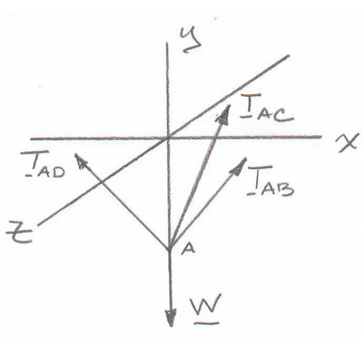
A crate is supported by three cables as shown. Determine the weight W of the crate, knowing that the tension in cable AD is 924 lb.

SOLUTION

The forces applied at A are:

$$\mathbf{T}_{AB}, \mathbf{T}_{AC}, \mathbf{T}_{AD} \text{ and } \mathbf{W}$$

where $\mathbf{P} = P\mathbf{j}$. To express the other forces in terms of the unit vectors **i**, **j**, **k**, we write



$$\overrightarrow{AB} = (28 \text{ in.})\mathbf{i} + (45 \text{ in.})\mathbf{j}$$

$$AB = 53 \text{ in.}$$

$$\overrightarrow{AC} = (45 \text{ in.})\mathbf{j} - (24 \text{ in.})\mathbf{k}$$

$$AC = 51 \text{ in.}$$

$$\overrightarrow{AD} = -(26 \text{ in.})\mathbf{i} + (45 \text{ in.})\mathbf{j} + (18 \text{ in.})\mathbf{k}$$

$$AD = 55 \text{ in.}$$

and

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB}$$
$$= (0.5283\mathbf{i} + 0.84906\mathbf{j})T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC}$$
$$= (0.88235\mathbf{j} - 0.47059\mathbf{k})T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD}$$
$$= (-0.47273\mathbf{i} + 0.81818\mathbf{j} + 0.32727\mathbf{k})T_{AD}$$

Equilibrium Condition with $\mathbf{W} = -W\mathbf{j}$

$$\Sigma F = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} - W\mathbf{j} = 0$$

PROBLEM 2.103 (Continued)

Substituting the expressions obtained for $\mathbf{T}_{AB}, \mathbf{T}_{AC}$, and $\mathbf{T}_{AD}$ and factoring **i**, **j**, and **k**:

$$(0.5283T_{AB} - 0.47273T_{AD})\mathbf{i} + (0.84906T_{AB} + 0.88235T_{AC} + 0.81818T_{AD} - W)\mathbf{j}$$
$$+ (-0.47059T_{AC} + 0.32727T_{AD})\mathbf{k} = 0$$

Equating to zero the coefficients of **i**, **j**, **k**:

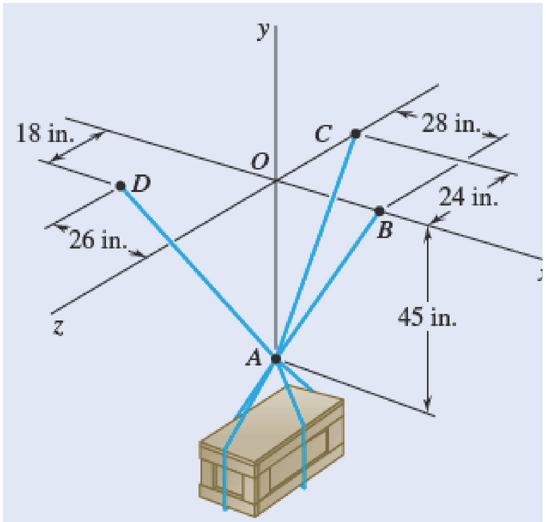
$$0.5283T_{AB} - 0.47273T_{AD} = 0 \quad (1)$$
$$0.84906T_{AB} + 0.88235T_{AC} + 0.81818T_{AD} - W = 0 \quad (2)$$
$$-0.47059T_{AC} + 0.32727T_{AD} = 0 \quad (3)$$

Substituting $T_{AD} = 924$ lb in Equations (1), (2), and (3) and solving the resulting set of equations, using conventional algorithms for solving linear algebraic equations, gives:

$$T_{AB} = 826.81 \text{ lb}$$
$$T_{AC} = 642.59 \text{ lb}$$

$W = 2030$ lb ◀

PROBLEM 2.104



A crate is supported by three cables as shown. Determine the weight W of the crate, knowing that the tension in cable AB is 1378 lb.

SOLUTION

See Problem 2.103 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$0.5283T_{AB} - 0.47273T_{AD} = 0 \quad (1)$$

$$0.84906T_{AB} + 0.88235T_{AC} + 0.81818T_{AD} - W = 0 \quad (2)$$

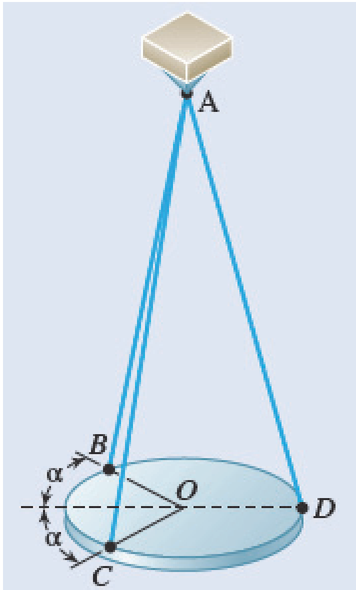
$$-0.47059T_{AC} + 0.32727T_{AD} = 0 \quad (3)$$

Substituting $T_{AB} = 1378 \text{ lb}$ in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives:

$$T_{AD} = 1539.99 \text{ lb}$$

$$T_{AC} = 1070.98 \text{ lb}$$

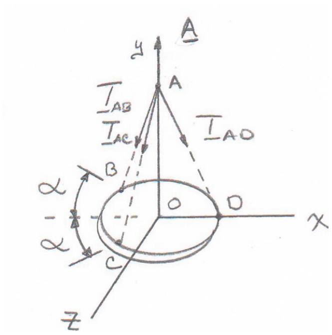
$W = 3380$ lb ◀



PROBLEM 2.105

A 12-lb circular plate of 7-in. radius is supported as shown by three wires, each of 25-in. length. Determine the tension in each wire, knowing that $\alpha = 30°$.

SOLUTION



Let θ be angle between the vertical and any wire.

$$OA = \sqrt{(25^2 - 7^2)} = 24 \text{ in. thus } \cos\theta = \frac{24}{25}$$

By symmetry $T_{AB} = T_{AC}$

$\Sigma F_x = 0$:

$$-2(T_{AB}\sin\theta)(\cos\alpha) + T_{AD}(\sin\theta) = 0$$

For $\alpha = 30°$:

$$T_{AD} = 2(\cos 30°)T_{AB} = 1.73205 T_{AB}$$

$\Sigma F_y = 0$: $\quad 12 \text{ lb} - 2T_{AB}(\cos\theta) - T_{AD}(\cos\theta) = 0$

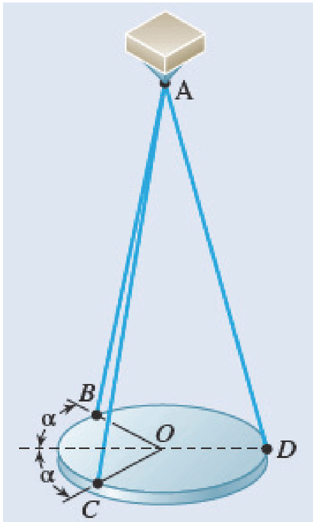
or $\quad 12 \text{ lb} = (2T_{AB} + T_{AD})\cos\theta$

$$12 \text{ lb} = (2T_{AB} + 1.73205 T_{AB})\left(\frac{24}{25}\right)$$

$T_{AB} = T_{AC} = 3.35$ lb ◀

$T_{AD} = 5.80$ lb ◀

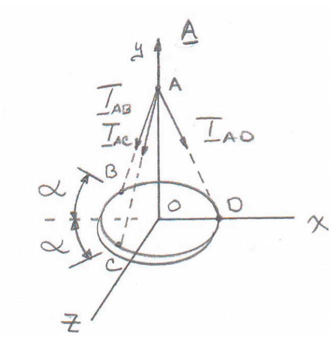
PROBLEM 2.106



Solve Prob. 2.105, knowing that $\alpha = 45°$.

PROBLEM 2.105 A 12-lb circular plate of 7-in. radius is supported as shown by three wires, each of 25-in. length. Determine the tension in each wire, knowing that $\alpha = 30°$.

SOLUTION



Let θ be angle between the vertical and any wire.

$$OA = \sqrt{(25^2 - 7^2)} = 24 \text{ in. thus } \cos\theta = \frac{24}{25}$$

By symmetry $T_{AB} = T_{AC}$

$\Sigma F_x = 0$:

$$-2(T_{AB}\sin\theta)(\cos\alpha) + T_{AD}(\sin\theta) = 0$$

For $\alpha = 45°$:

$$T_{AD} = 2(\cos 45°)T_{AB} = 1.41421 T_{AB}$$

$\Sigma F_y = 0$: $\quad 12 \text{ lb} - 2T_{AB}(\cos\theta) - T_{AD}(\cos\theta) = 0$

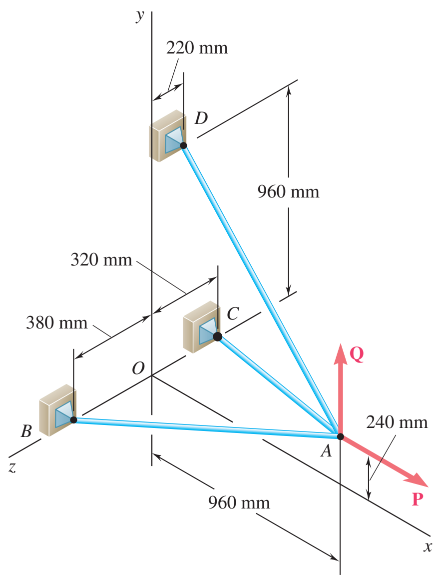
or $$12 \text{ lb} = (2T_{AB} + T_{AD})\cos\theta$$

$$12 \text{ lb} = (2T_{AB} + 1.41421 T_{AB})\left(\frac{24}{25}\right)$$

$$T_{AB} = T_{AC} = 3.66 \text{ lb} \blacktriangleleft$$

$$T_{AD} = 5.18 \text{ lb} \blacktriangleleft$$

PROBLEM 2.107



Three cables are connected at A, where the forces **P** and **Q** are applied as shown. Knowing that $Q = 0$, find the value of P for which the tension in cable AD is 305 N.

SOLUTION

$$\Sigma \mathbf{F}_A = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{P} = 0 \quad \text{where} \quad \mathbf{P} = P\mathbf{i}$$

$$\overrightarrow{AB} = -(960\text{ mm})\mathbf{i} - (240\text{ mm})\mathbf{j} + (380\text{ mm})\mathbf{k} \qquad AB = 1060\text{ mm}$$

$$\overrightarrow{AC} = -(960\text{ mm})\mathbf{i} - (240\text{ mm})\mathbf{j} - (320\text{ mm})\mathbf{k} \qquad AC = 1040\text{ mm}$$

$$\overrightarrow{AD} = -(960\text{ mm})\mathbf{i} + (720\text{ mm})\mathbf{j} - (220\text{ mm})\mathbf{k} \qquad AD = 1220\text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = T_{AB}\left(-\frac{48}{53}\mathbf{i} - \frac{12}{53}\mathbf{j} + \frac{19}{53}\mathbf{k}\right)$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = T_{AC}\left(-\frac{12}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} - \frac{4}{13}\mathbf{k}\right)$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = \frac{305\text{ N}}{1220\text{ mm}}[(-960\text{ mm})\mathbf{i} + (720\text{ mm})\mathbf{j} - (220\text{ mm})\mathbf{k}]$$

$$= -(240\text{ N})\mathbf{i} + (180\text{ N})\mathbf{j} - (55\text{ N})\mathbf{k}$$

Substituting into $\Sigma \mathbf{F}_A = 0$, factoring **i**, **j**, **k**, and setting each coefficient equal to ϕ gives:

$$\mathbf{i}: \quad P = \frac{48}{53}T_{AB} + \frac{12}{13}T_{AC} + 240\text{ N} \qquad (1)$$

$$\mathbf{j}: \quad \frac{12}{53}T_{AB} + \frac{3}{13}T_{AC} = 180\text{ N} \qquad (2)$$

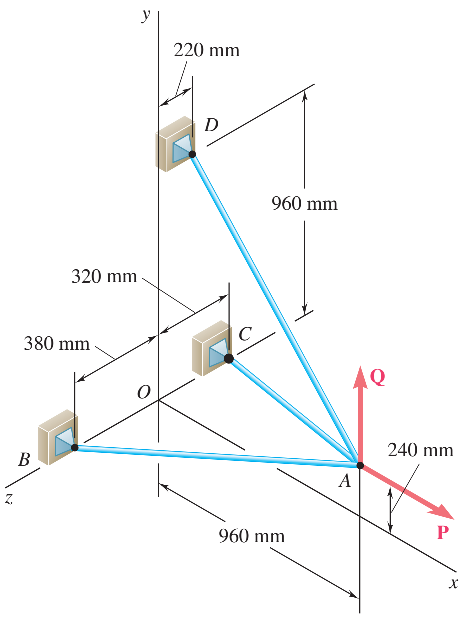
$$\mathbf{k}: \quad \frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} = 55\text{ N} \qquad (3)$$

Solving the system of linear equations using conventional algorithms gives:

$$T_{AB} = 446.71\text{ N}$$

$$T_{AC} = 341.71\text{ N}$$

$P = 960$ N ◀



PROBLEM 2.108

Three cables are connected at A, where the forces **P** and **Q** are applied as shown. Knowing that $P = 1200$ N, determine the values of Q for which cable AD is taut.

SOLUTION

We assume that $T_{AD} = 0$ and write $\quad \Sigma \mathbf{F}_A = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + Q\mathbf{j} + (1200 \text{ N})\mathbf{i} = 0$

$$\overrightarrow{AB} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} + (380 \text{ mm})\mathbf{k} \quad AB = 1060 \text{ mm}$$

$$\overrightarrow{AC} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k} \quad AC = 1040 \text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \left(-\frac{48}{53}\mathbf{i} - \frac{12}{53}\mathbf{j} + \frac{19}{53}\mathbf{k}\right)T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \left(-\frac{12}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} - \frac{4}{13}\mathbf{k}\right)T_{AC}$$

Substituting into $\Sigma \mathbf{F}_A = 0$, factoring **i**, **j**, **k**, and setting each coefficient equal to ϕ gives:

$$\mathbf{i}: \quad -\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} + 1200 \text{ N} = 0 \qquad (1)$$

$$\mathbf{j}: \quad -\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + Q = 0 \qquad (2)$$

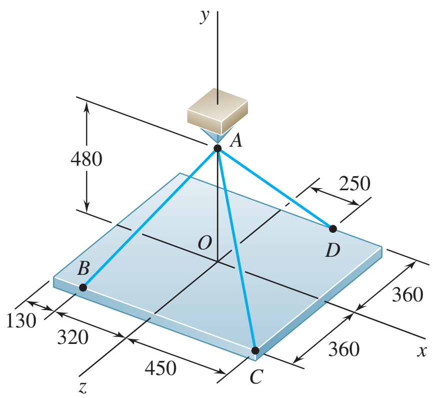
$$\mathbf{k}: \quad \frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} = 0 \qquad (3)$$

Solving the resulting system of linear equations using conventional algorithms gives:

$$T_{AB} = 605.71 \text{ N}$$
$$T_{AC} = 705.71 \text{ N}$$
$$Q = 300.00 \text{ N} \qquad 0 \le Q < 300 \text{ N} \blacktriangleleft$$

Note: This solution assumes that Q is directed upward as shown $(Q \geq 0)$, if negative values of Q are considered, cable AD remains taut, but AC becomes slack for $Q = -460$ N.

PROBLEM 2.109



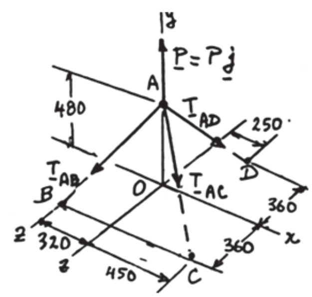
Dimensions in mm

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable AC is 60 N, determine the weight of the plate.

SOLUTION

We note that the weight of the plate is equal in magnitude to the force **P** exerted by the support on Point A.

Free Body A:



Dimensions in mm

$$\Sigma F = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$$

We have:

$$\overrightarrow{AB} = -(320 \text{ mm})\mathbf{i} - (480 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k} \qquad AB = 680 \text{ mm}$$

$$\overrightarrow{AC} = (450 \text{ mm})\mathbf{i} - (480 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k} \qquad AC = 750 \text{ mm}$$

$$\overrightarrow{AD} = (250 \text{ mm})\mathbf{i} - (480 \text{ mm})\mathbf{j} - \left(360 \text{ mm}\right)\mathbf{k} \qquad AD = 650 \text{ mm}$$

Thus:

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \left(-\frac{8}{17}\mathbf{i} - \frac{12}{17}\mathbf{j} + \frac{9}{17}\mathbf{k}\right)T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \left(0.6\mathbf{i} - 0.64\mathbf{j} + 0.48\mathbf{k}\right)T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = \left(\frac{5}{13}\mathbf{i} - \frac{9.6}{13}\mathbf{j} - \frac{7.2}{13}\mathbf{k}\right)T_{AD}$$

Substituting into the Eq. $\Sigma F = 0$ and factoring $\mathbf{i}, \mathbf{j}, \mathbf{k}$:

$$\left(-\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD}\right)\mathbf{i}$$
$$+\left(-\frac{12}{17}T_{AB} - 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P\right)\mathbf{j}$$
$$+\left(\frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD}\right)\mathbf{k} = 0$$

PROBLEM 2.109 (Continued)

Setting the coefficient of **i**, **j**, **k** equal to zero:

$$\mathbf{i}: \quad -\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} = 0 \qquad (1)$$

$$\mathbf{j}: \quad -\frac{12}{7}T_{AB} - 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P = 0 \qquad (2)$$

$$\mathbf{k}: \quad \frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} = 0 \qquad (3)$$

Making $T_{AC} = 60$ N in (1) and (3):

$$-\frac{8}{17}T_{AB} + 36\text{ N} + \frac{5}{13}T_{AD} = 0 \qquad (1')$$

$$\frac{9}{17}T_{AB} + 28.8\text{ N} - \frac{7.2}{13}T_{AD} = 0 \qquad (3')$$

Multiply (1′) by 9, (3′) by 8, and add:

$$554.4\text{ N} - \frac{12.6}{13}T_{AD} = 0 \quad T_{AD} = 572.0\text{ N}$$

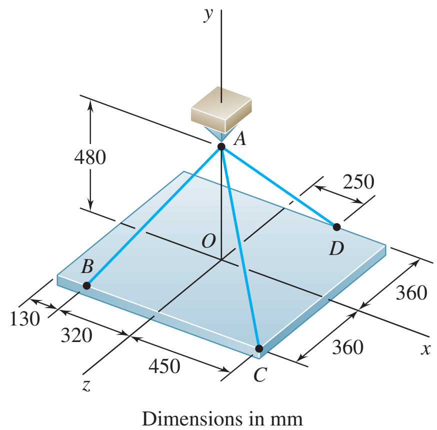
Substitute into (1′) and solve for T_{AB}:

$$T_{AB} = \frac{17}{8}\left(36 + \frac{5}{13}\times 572\right) \qquad T_{AB} = 544.0\text{ N}$$

Substitute for the tensions in Eq. (2) and solve for P:

$$P = \frac{12}{17}(544\text{ N}) + 0.64(60\text{ N}) + \frac{9.6}{13}(572\text{ N})$$
$$= 844.8\text{ N}$$

Weight of plate $= P = 845$ N ◀



PROBLEM 2.110

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable *AD* is 520 N, determine the weight of the plate.

SOLUTION

See Problem 2.109 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{8}{17}T_{AB}+0.6T_{AC}+\frac{5}{13}T_{AD}=0 \qquad (1)$$

$$-\frac{12}{17}T_{AB}+0.64T_{AC}-\frac{9.6}{13}T_{AD}+P=0 \qquad (2)$$

$$\frac{9}{17}T_{AB}+0.48T_{AC}-\frac{7.2}{13}T_{AD}=0 \qquad (3)$$

Making $T_{AD}=520\text{ N}$ in Eqs. (1) and (3):

$$-\frac{8}{17}T_{AB}+0.6T_{AC}+200\text{ N}=0 \qquad (1')$$

$$\frac{9}{17}T_{AB}+0.48T_{AC}-288\text{ N}=0 \qquad (3')$$

Multiply (1′) by 9, (3′) by 8, and add:

$$9.24T_{AC}-504\text{ N}=0 \quad T_{AC}=54.5455\text{ N}$$

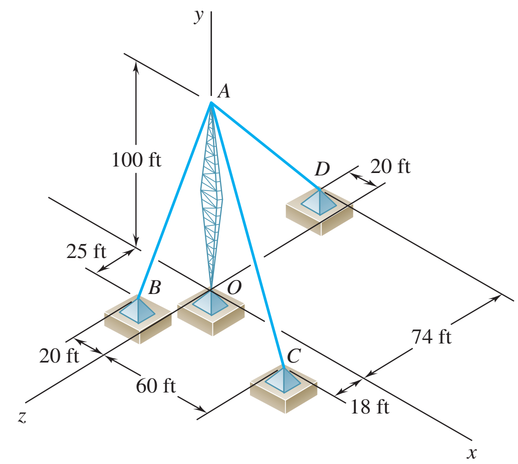
Substitute into (1′) and solve for T_{AB}:

$$T_{AB}=\frac{17}{8}(0.6\times 54.5455+200) \quad T_{AB}=494.545\text{ N}$$

Substitute for the tensions in Eq. (2) and solve for P:

$$P=\frac{12}{17}(494.545\text{ N})+0.64(54.5455\text{ N})+\frac{9.6}{13}(520\text{ N})$$
$$=768.00\text{ N}$$

Weight of plate $=P=768\text{ N}$ ◀



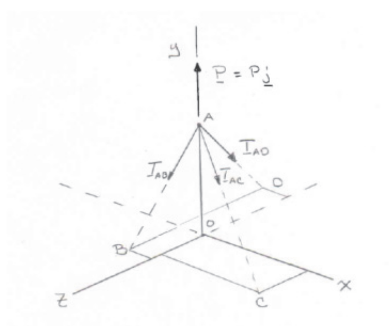
PROBLEM 2.111

A transmission tower is held by three guy wires attached to a pin at A and anchored by bolts at B, C, and D. If the tension in wire AB is 840 lb, determine the vertical force $\mathbf{P}$ exerted by the tower on the pin at A.

SOLUTION

$$\Sigma\mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$$

Free-Body Diagram at A:



$$\overrightarrow{AB} = -20\mathbf{i} - 100\mathbf{j} + 25\mathbf{k} \quad AB = 105 \text{ ft}$$
$$\overrightarrow{AC} = 60\mathbf{i} - 100\mathbf{j} + 18\mathbf{k} \quad AC = 118 \text{ ft}$$
$$\overrightarrow{AD} = -20\mathbf{i} - 100\mathbf{j} - 74\mathbf{k} \quad AD = 126 \text{ ft}$$

We write

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \left(-\frac{4}{21}\mathbf{i} - \frac{20}{21}\mathbf{j} + \frac{5}{21}\mathbf{k}\right)T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \left(\frac{30}{59}\mathbf{i} - \frac{50}{59}\mathbf{j} + \frac{9}{59}\mathbf{k}\right)T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = \left(-\frac{10}{63}\mathbf{i} - \frac{50}{63}\mathbf{j} - \frac{37}{63}\mathbf{k}\right)T_{AD}$$

Substituting into the Eq. $\Sigma\mathbf{F} = 0$ and factoring $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$:

PROBLEM 2.111 (Continued)

$$\left(-\frac{4}{21}T_{AB}+\frac{30}{59}T_{AC}-\frac{10}{63}T_{AD}\right)\mathbf{i}$$
$$+\left(-\frac{20}{21}T_{AB}-\frac{50}{59}T_{AC}-\frac{50}{63}T_{AD}+P\right)\mathbf{j}$$
$$+\left(\frac{5}{21}T_{AB}+\frac{9}{59}T_{AC}-\frac{37}{63}T_{AD}\right)\mathbf{k}=0$$

Setting the coefficients of $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, equal to zero:

$$\mathbf{i}: \quad -\frac{4}{21}T_{AB}+\frac{30}{59}T_{AC}-\frac{10}{63}T_{AD}=0 \qquad (1)$$

$$\mathbf{j}: \quad -\frac{20}{21}T_{AB}-\frac{50}{59}T_{AC}-\frac{50}{63}T_{AD}+P=0 \qquad (2)$$

$$\mathbf{k}: \quad \frac{5}{21}T_{AB}+\frac{9}{59}T_{AC}-\frac{37}{63}T_{AD}=0 \qquad (3)$$

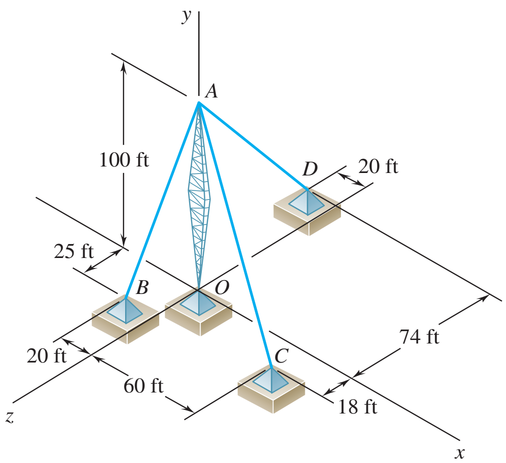
Set $T_{AB}=840$ lb in Eqs. (1) – (3):

$$-160\text{ lb}+\frac{30}{59}T_{AC}-\frac{10}{63}T_{AD}=0 \qquad (1')$$

$$-800\text{ lb}-\frac{50}{59}T_{AC}-\frac{50}{63}T_{AD}+P=0 \qquad (2')$$

$$200\text{ lb}+\frac{9}{59}T_{AC}-\frac{37}{63}T_{AD}=0 \qquad (3')$$

Solving, $T_{AC}=458.12$ lb $T_{AD}=459.53$ lb $P=1552.94$ lb $\qquad P=1553$ lb ◀



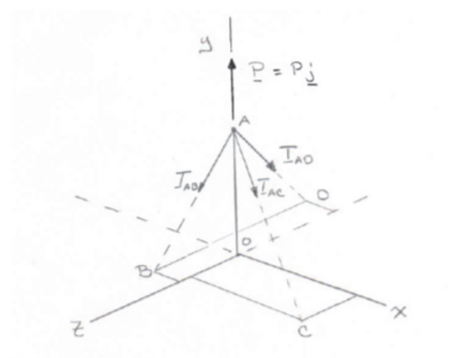
PROBLEM 2.112

A transmission tower is held by three guy wires attached to a pin at A and anchored by bolts at B, C, and D. If the tension in wire AC is 590 lb, determine the vertical force $\mathbf{P}$ exerted by the tower on the pin at A.

SOLUTION

$$\Sigma\mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$$

Free-Body Diagram at A:



$$\overrightarrow{AB} = -20\mathbf{i} - 100\mathbf{j} + 25\mathbf{k} \qquad AB = 105 \text{ ft}$$

$$\overrightarrow{AC} = 60\mathbf{i} - 100\mathbf{j} + 18\mathbf{k} \qquad AC = 118 \text{ ft}$$

$$\overrightarrow{AD} = -20\mathbf{i} - 100\mathbf{j} - 74\mathbf{k} \qquad AD = 126 \text{ ft}$$

We write

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB}$$

$$= \left(-\frac{4}{21}\mathbf{i} - \frac{20}{21}\mathbf{j} + \frac{5}{21}\mathbf{k}\right)T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC}$$

$$= \left(\frac{30}{59}\mathbf{i} - \frac{50}{59}\mathbf{j} + \frac{9}{59}\mathbf{k}\right)T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD}$$

$$= \left(-\frac{10}{63}\mathbf{i} - \frac{50}{63}\mathbf{j} - \frac{37}{63}\mathbf{k}\right)T_{AD}$$

Substituting into the Eq. $\Sigma\mathbf{F} = 0$ and factoring $\mathbf{i}, \mathbf{j}, \mathbf{k}$:

$$\left(-\frac{4}{21}T_{AB} + \frac{30}{59}T_{AC} - \frac{10}{63}T_{AD}\right)\mathbf{i}$$
$$+\left(-\frac{20}{21}T_{AB} - \frac{50}{59}T_{AC} - \frac{50}{63}T_{AD} + P\right)\mathbf{j}$$
$$+\left(\frac{5}{21}T_{AB} + \frac{9}{59}T_{AC} - \frac{37}{63}T_{AD}\right)\mathbf{k} = 0$$

Setting the coefficients of **i**, **j**, **k**, equal to zero:

$$\mathbf{i}: \quad -\frac{4}{21}T_{AB} + \frac{30}{59}T_{AC} - \frac{10}{63}T_{AD} = 0 \tag{1}$$

$$\mathbf{j}: \quad -\frac{20}{21}T_{AB} - \frac{50}{59}T_{AC} - \frac{50}{63}T_{AD} + P = 0 \tag{2}$$

$$\mathbf{k}: \quad \frac{5}{21}T_{AB} + \frac{9}{59}T_{AC} - \frac{37}{63}T_{AD} = 0 \tag{3}$$

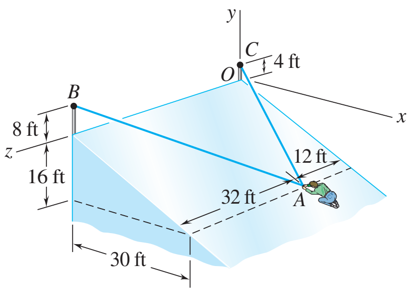
Set $T_{AC} = 590$ lb in Eqs. (1) – (3):

$$-\frac{4}{21}T_{AB} + 300\text{ lb} - \frac{10}{63}T_{AD} = 0 \tag{1'}$$

$$-\frac{20}{21}T_{AB} - 500\text{ lb} - \frac{50}{63}T_{AD} + P = 0 \tag{2'}$$

$$\frac{5}{21}T_{AB} + 90\text{ lb} - \frac{37}{63}T_{AD} = 0 \tag{3'}$$

Solving, $T_{AB} = 1081.82$ lb $\quad T_{AD} = 591.82$ lb $\qquad P = 2000$ lb ◀

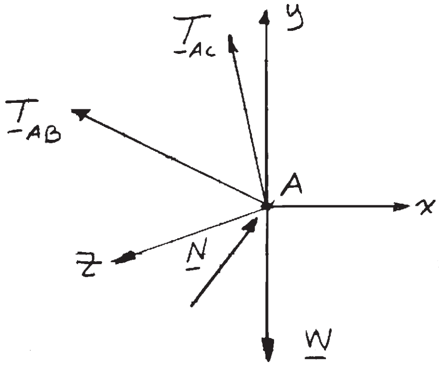


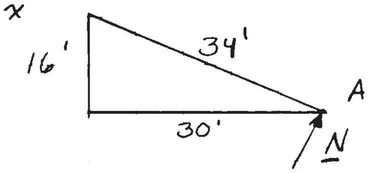
PROBLEM 2.113

In trying to move across a slippery icy surface, a 175-lb man uses two ropes *AB* and *AC*. Knowing that the force exerted on the man by the icy surface is perpendicular to that surface, determine the tension in each rope.

SOLUTION

Free-Body Diagram at *A*





$$\mathbf{N} = N\left(\frac{16}{34}\mathbf{i} + \frac{30}{34}\mathbf{j}\right)$$

and $\mathbf{W} = W\ \mathbf{j} = -(175\text{ lb})\mathbf{j}$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = T_{AC}\frac{(-30\text{ ft})\mathbf{i} + (20\text{ ft})\mathbf{j} - (12\text{ ft})\mathbf{k}}{38\text{ ft}}$$

$$= T_{AC}\left(-\frac{15}{19}\mathbf{i} + \frac{10}{19}\mathbf{j} - \frac{6}{19}\mathbf{k}\right)$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = T_{AB}\frac{(-30\text{ ft})\mathbf{i} + (24\text{ ft})\mathbf{j} + (32\text{ ft})\mathbf{k}}{50\text{ ft}}$$

$$= T_{AB}\left(-\frac{15}{25}\mathbf{i} + \frac{12}{25}\mathbf{j} + \frac{16}{25}\mathbf{k}\right)$$

Equilibrium condition: $\Sigma\mathbf{F} = 0$

$$\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{N} + \mathbf{W} = 0$$

PROBLEM 2.113 (Continued)

Substituting the expressions obtained for $\mathbf{T}_{AB}, \mathbf{T}_{AC}, \mathbf{N},$ and **W**; factoring **i**, **j**, and **k**; and equating each of the coefficients to zero gives the following equations:

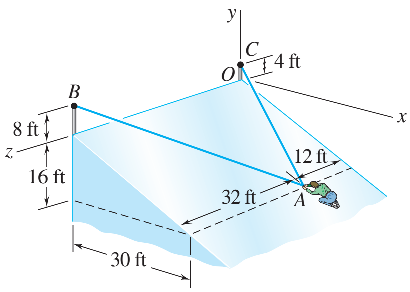
From **i**:
$$-\frac{15}{25}T_{AB} - \frac{15}{19}T_{AC} + \frac{16}{34}N = 0 \qquad (1)$$

From **j**:
$$\frac{12}{25}T_{AB} + \frac{10}{19}T_{AC} + \frac{30}{34}N - (175 \text{ lb}) = 0 \qquad (2)$$

From **k**:
$$\frac{16}{25}T_{AB} - \frac{6}{19}T_{AC} = 0 \qquad (3)$$

Solving the resulting set of equations gives:

$T_{AB} = 30.8$ lb; $T_{AC} = 62.5$ lb ◀



PROBLEM 2.114

Solve Problem 2.113, assuming that a friend is helping the man at A by pulling on him with a force $\mathbf{P} = -(45 \text{ lb})\mathbf{k}$.

PROBLEM 2.113 In trying to move across a slippery icy surface, a 175-lb man uses two ropes AB and AC. Knowing that the force exerted on the man by the icy surface is perpendicular to that surface, determine the tension in each rope.

SOLUTION

Refer to Problem 2.113 for the figure and analysis leading to the following set of equations, Equation (3) being modified to include the additional force $\mathbf{P} = (-45 \text{ lb})\mathbf{k}$.

$$-\frac{15}{25}T_{AB} - \frac{15}{19}T_{AC} + \frac{16}{34}N = 0 \qquad (1)$$

$$\frac{12}{25}T_{AB} + \frac{10}{19}T_{AC} + \frac{30}{34}N - (175 \text{ lb}) = 0 \qquad (2)$$

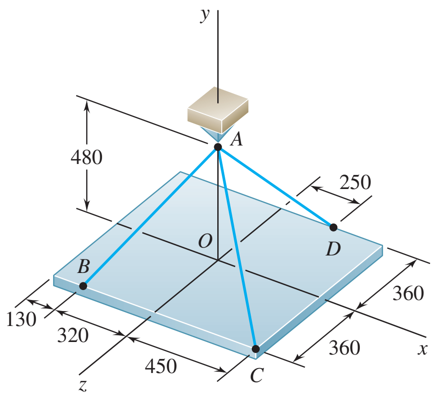
$$\frac{16}{25}T_{AB} - \frac{6}{19}T_{AC} - (45 \text{ lb}) = 0 \qquad (3)$$

Solving the resulting set of equations simultaneously gives:

$T_{AB} = 81.3 \text{ lb}$ ◀

$T_{AC} = 22.2 \text{ lb}$ ◀

PROBLEM 2.115



For the rectangular plate of Problems 2.109 and 2.110, determine the tension in each of the three cables knowing that the weight of the plate is 792 N.

SOLUTION

See Problem 2.109 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below. Setting $P = 792$ N gives:

$$-\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} = 0 \quad (1)$$

$$-\frac{12}{17}T_{AB} - 0.64T_{AC} - \frac{9.6}{13}T_{AD} + 792\text{ N} = 0 \quad (2)$$

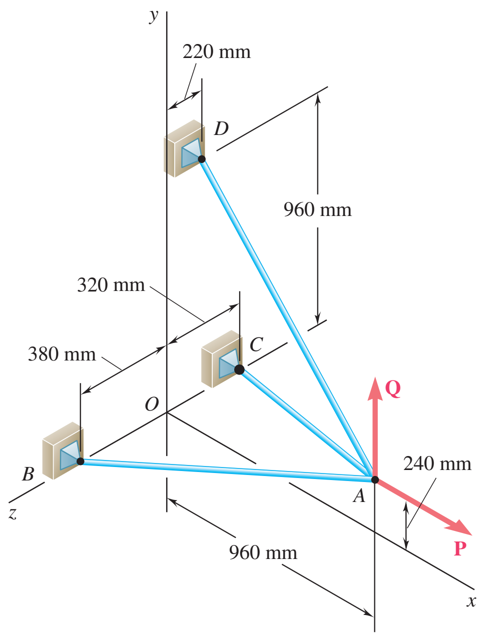
$$\frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} = 0 \quad (3)$$

Solving Equations (1), (2), and (3) by conventional algorithms gives

$T_{AB} = 510.00$ N $\qquad T_{AB} = 510$ N ◀

$T_{AC} = 56.250$ N $\qquad T_{AC} = 56.2$ N ◀

$T_{AD} = 536.25$ N $\qquad T_{AD} = 536$ N ◀



PROBLEM 2.116

For the cable system of Problems 2.107 and 2.108, determine the tension in each cable knowing that $P = 2880$ N and $Q = 0$.

SOLUTION

$$\Sigma \mathbf{F}_A = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{P} + \mathbf{Q} = 0$$

Where $\quad \mathbf{P} = P\mathbf{i}$ and $\mathbf{Q} = Q\mathbf{j}$

$$\overrightarrow{AB} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} + (380 \text{ mm})\mathbf{k} \quad AB = 1060 \text{ mm}$$

$$\overrightarrow{AC} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k} \quad AC = 1040 \text{ mm}$$

$$\overrightarrow{AD} = -(960 \text{ mm})\mathbf{i} + (720 \text{ mm})\mathbf{j} - (220 \text{ mm})\mathbf{k} \quad AD = 1220 \text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = T_{AB}\left(-\frac{48}{53}\mathbf{i} - \frac{12}{53}\mathbf{j} + \frac{19}{53}\mathbf{k}\right)$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = T_{AC}\left(-\frac{12}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} - \frac{4}{13}\mathbf{k}\right)$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = T_{AD}\left(-\frac{48}{61}\mathbf{i} + \frac{36}{61}\mathbf{j} - \frac{11}{61}\mathbf{k}\right)$$

Substituting into $\Sigma \mathbf{F}_A = 0$, setting $P = (2880 \text{ N})\mathbf{i}$ and $Q = 0$, and setting the coefficients of **i**, **j**, **k** equal to 0, we obtain the following three equilibrium equations:

$$\mathbf{i}: \quad -\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + 2880 \text{ N} = 0 \qquad (1)$$

$$\mathbf{j}: \quad -\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} = 0 \qquad (2)$$

$$\mathbf{k}: \quad \frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \qquad (3)$$

PROBLEM 2.116 (Continued)

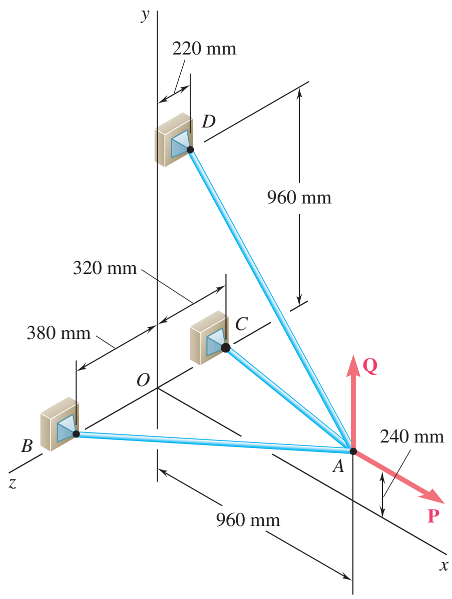
Solving the system of linear equations using conventional algorithms gives:

$$T_{AB} = 1340.14 \text{ N}$$
$$T_{AC} = 1025.12 \text{ N}$$
$$T_{AD} = 915.03 \text{ N}$$

$T_{AB} = 1340 \text{ N}$ ◀

$T_{AC} = 1025 \text{ N}$ ◀

$T_{AD} = 915 \text{ N}$ ◀



PROBLEM 2.117

For the cable system of Problems 2.107 and 2.108, determine the tension in each cable knowing that $P = 2880$ N and $Q = 576$ N.

SOLUTION

See Problem 2.116 for the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + P = 0 \tag{1}$$

$$-\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} + Q = 0 \tag{2}$$

$$\frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \tag{3}$$

Setting $P = 2880$ N and $Q = 576$ N gives:

$$-\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + 2880\text{ N} = 0 \tag{1'}$$

$$-\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} + 576\text{ N} = 0 \tag{2'}$$

$$\frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \tag{3'}$$

Solving the resulting set of equations using conventional algorithms gives:

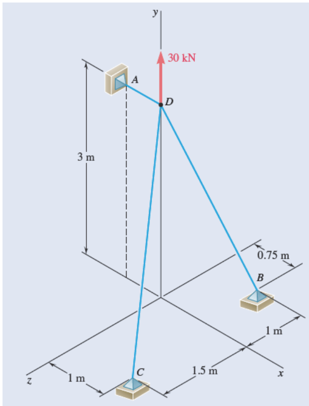
$$T_{AB} = 1431.00\text{ N}$$
$$T_{AC} = 1560.00\text{ N}$$
$$T_{AD} = 183.010\text{ N}$$

$T_{AB} = 1431$ N ◀

$T_{AC} = 1560$ N ◀

$T_{AD} = 183.0$ N ◀

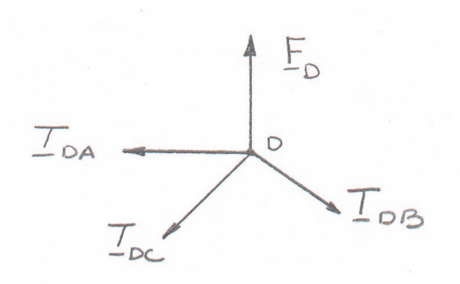
PROBLEM 2.118



Three cables are connected at D, where an upward force of 30 kN is applied. Determine the tension in each cable.

SOLUTION

Free-Body Diagram of Point D:



$\Sigma \mathbf{F}_D = 0$: $\quad \mathbf{T}_{DA} + \mathbf{T}_{DB} + \mathbf{T}_{DC} + \mathbf{P} = 0$

Where $\quad \mathbf{P} = P\mathbf{j}$

$$\overrightarrow{DA} = -(1 \text{ m})\mathbf{i} \quad DA = 1 \text{ m}$$

$$\overrightarrow{DB} = +(0.75 \text{ m})\mathbf{i} - (3 \text{ m})\mathbf{j} - (1 \text{ m})\mathbf{k} \quad DB = 3.25 \text{ m}$$

$$\overrightarrow{DC} = +(1 \text{ m})\mathbf{i} - (3 \text{ m})\mathbf{j} + (1.5 \text{ m})\mathbf{k} \quad DC = 3.5 \text{ m}$$

$$\mathbf{T}_{DA} = T_{DA}\boldsymbol{\lambda}_{DA} = T_{DA}\frac{\overrightarrow{DA}}{DA} = T_{DA}(-\mathbf{i})$$

$$\mathbf{T}_{DB} = T_{DB}\boldsymbol{\lambda}_{DB} = T_{DB}\frac{\overrightarrow{DB}}{DB} = T_{DB}\left(\frac{3}{13}\mathbf{i} - \frac{12}{13}\mathbf{j} - \frac{4}{13}\mathbf{k}\right)$$

$$\mathbf{T}_{DC} = T_{DC}\boldsymbol{\lambda}_{DC} = T_{DC}\frac{\overrightarrow{DC}}{DC} = T_{DC}\left(\frac{2}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}\right)$$

Substituting into $\Sigma \mathbf{F}_D = 0$, setting $P = (30 \text{ kN})\mathbf{j}$ and setting the coefficients of $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ equal to 0, we obtain the following three equilibrium equations:

$$\mathbf{i}: \quad -T_{DA} + \frac{3}{13}T_{DB} + \frac{2}{7}T_{DC} = 0 \qquad (1)$$

PROBLEM 2.118 (Continued)

$$\mathbf{j}: \quad -\frac{12}{13}T_{DB} - \frac{6}{7}T_{DC} + 30 \text{ kN} = 0 \quad (2)$$

$$\mathbf{k}: \quad -\frac{4}{13}T_{DB} + \frac{3}{7}T_{DC} = 0 \quad (3)$$

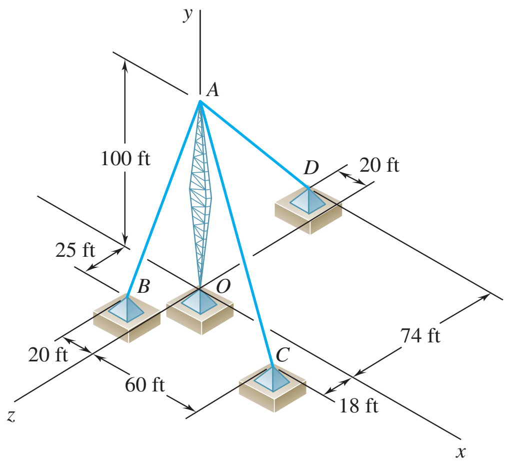
Solving the system of linear equations using conventional algorithms gives:

$T_{DA} = 8.50 \text{ kN}$ ◀

$T_{DB} = 19.50 \text{ kN}$ ◀

$T_{DC} = 14.00 \text{ kN}$ ◀

PROBLEM 2.119



For the transmission tower of Probs. 2.111 and 2.112, determine the tension in each guy wire knowing that the tower exerts on the pin at A an upward vertical force of 1800 lb.

PROBLEM 2.111 A transmission tower is held by three guy wires attached to a pin at A and anchored by bolts at B, C, and D. If the tension in wire AB is 840 lb, determine the vertical force $\mathbf{P}$ exerted by the tower on the pin at A.

SOLUTION

See Problem 2.111 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$\mathbf{i}: \quad -\frac{4}{21}T_{AB} + \frac{30}{59}T_{AC} - \frac{10}{63}T_{AD} = 0 \qquad (1)$$

$$\mathbf{j}: \quad -\frac{20}{21}T_{AB} - \frac{50}{59}T_{AC} - \frac{50}{63}T_{AD} + P = 0 \qquad (2)$$

$$\mathbf{k}: \quad \frac{5}{21}T_{AB} + \frac{9}{59}T_{AC} - \frac{37}{63}T_{AD} = 0 \qquad (3)$$

Substituting for $P = 1800$ lb in Equations (1), (2), and (3) above and solving the resulting set of equations using conventional algorithms gives:

$$-\frac{4}{21}T_{AB} + \frac{30}{59}T_{AC} - \frac{10}{63}T_{AD} = 0 \qquad (1')$$

$$-\frac{20}{21}T_{AB} - \frac{50}{59}T_{AC} - \frac{50}{63}T_{AD} + 1800 \text{ lb} = 0 \qquad (2')$$

$$\frac{5}{21}T_{AB} + \frac{9}{59}T_{AC} - \frac{37}{63}T_{AD} = 0 \qquad (3')$$

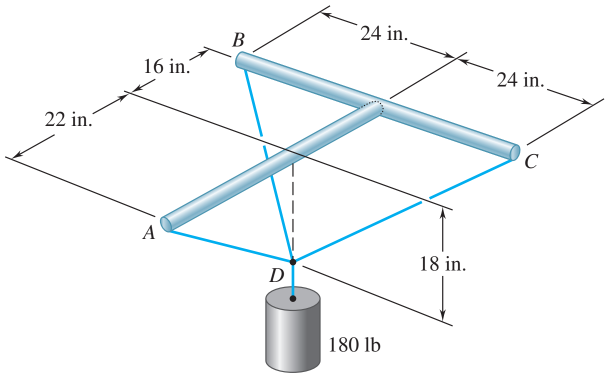
$T_{AB} = 973.64$ lb
$T_{AC} = 531.00$ lb
$T_{AD} = 532.64$ lb

$T_{AB} = 974$ lb ◀

$T_{AC} = 531$ lb ◀

$T_{AD} = 533$ lb ◀

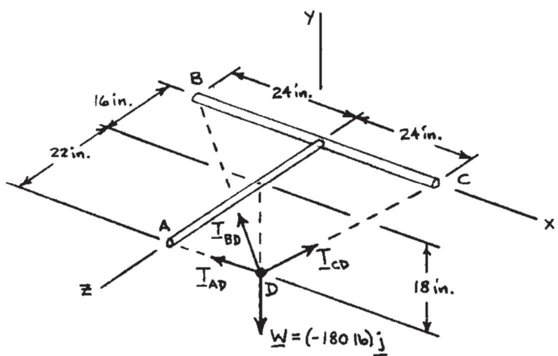
PROBLEM 2.120



Three wires are connected at point D, which is located 18 in. below the T-shaped pipe support ABC. Determine the tension in each wire when a 180-lb cylinder is suspended from point D as shown.

SOLUTION

Free-Body Diagram of Point D:



The forces applied at D are:

$$\mathbf{T}_{DA}, \mathbf{T}_{DB}, \mathbf{T}_{DC} \text{ and } \mathbf{W}$$

where $\mathbf{W} = -180.0 \text{ lb}\mathbf{j}$. To express the other forces in terms of the unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, we write

$$\overrightarrow{DA} = (18 \text{ in.})\mathbf{j} + (22 \text{ in.})\mathbf{k}$$

$$DA = 28.425 \text{ in.}$$

$$\overrightarrow{DB} = -(24 \text{ in.})\mathbf{i} + (18 \text{ in.})\mathbf{j} - (16 \text{ in.})\mathbf{k}$$

$$DB = 34.0 \text{ in.}$$

$$\overrightarrow{DC} = (24 \text{ in.})\mathbf{i} + (18 \text{ in.})\mathbf{j} - (16 \text{ in.})\mathbf{k}$$

$$DC = 34.0 \text{ in.}$$

PROBLEM 2.120 (Continued)

and
$$\mathbf{T}_{DA} = T_{Da}\boldsymbol{\lambda}_{DA} = T_{Da}\frac{\overrightarrow{DA}}{DA}$$
$$= (0.63324\mathbf{j} + 0.77397\mathbf{k})T_{DA}$$
$$\mathbf{T}_{DB} = T_{DB}\boldsymbol{\lambda}_{DB} = T_{DB}\frac{\overrightarrow{DB}}{DB}$$
$$= (-0.70588\mathbf{i} + 0.52941\mathbf{j} - 0.47059\mathbf{k})T_{DB}$$
$$\mathbf{T}_{DC} = T_{DC}\boldsymbol{\lambda}_{DC} = T_{DC}\frac{\overrightarrow{DC}}{DC}$$
$$= (0.70588\mathbf{i} + 0.52941\mathbf{j} - 0.47059\mathbf{k})T_{DC}$$

Equilibrium Condition with $\mathbf{W} = -W\mathbf{j}$

$$\Sigma F = 0: \quad \mathbf{T}_{DA} + \mathbf{T}_{DB} + \mathbf{T}_{DC} - W\mathbf{j} = 0$$

Substituting the expressions obtained for $\mathbf{T}_{DA}$, $\mathbf{T}_{DB}$, and $\mathbf{T}_{DC}$ and factoring **i**, **j**, and **k**:

$$(-0.70588T_{DB} + 0.70588T_{DC})\mathbf{i}$$
$$(0.63324T_{DA} + 0.52941T_{DB} + 0.52941T_{DC} - W)\mathbf{j}$$
$$(0.77397T_{DA} - 0.47059T_{DB} - 0.47059T_{DC})\mathbf{k}$$

Equating to zero the coefficients of **i**, **j**, **k**:

$$-0.70588T_{DB} + 0.70588T_{DC} = 0 \qquad (1)$$

$$0.63324T_{DA} + 0.52941T_{DB} + 0.52941T_{DC} - W = 0 \qquad (2)$$

$$0.77397T_{DA} - 0.47059T_{DB} - 0.47059T_{DC} = 0 \qquad (3)$$

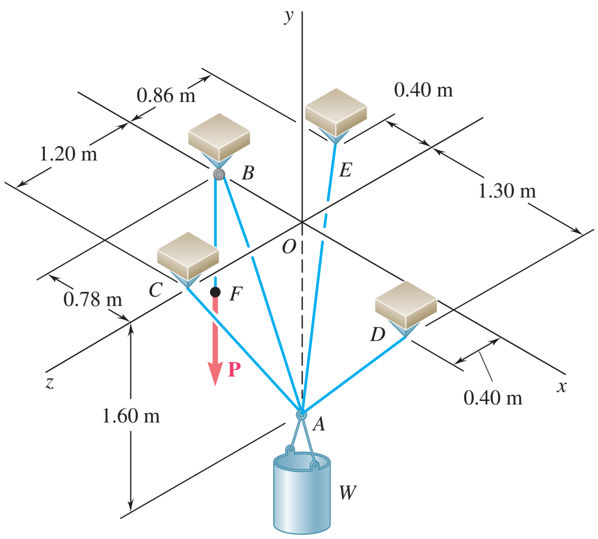
Substituting $W = 180$ lb in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms gives,

$T_{DA} = 119.7$ lb ◀

$T_{DB} = 98.4$ lb ◀

$T_{DC} = 98.4$ lb ◀

PROBLEM 2.121



A container of weight W is suspended from ring A, to which cables AC and AE are attached. A force $\mathbf{P}$ is applied to the end F of a third cable that passes over a pulley at B and through ring A and that is attached to a support at D. Knowing that $W = 1000$ N, determine the magnitude of P. (*Hint:* The tension is the same in all portions of cable $FBAD$.)

SOLUTION

The (vector) force in each cable can be written as the product of the (scalar) force and the unit vector along the cable. That is, with

$$\overrightarrow{AB} = -(0.78 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j} + (0 \text{ m})\mathbf{k}$$

$$AB = \sqrt{(-0.78 \text{ m})^2 + (1.6 \text{ m})^2 + (0)^2}$$
$$= 1.78 \text{ m}$$

$$\mathbf{T}_{AB} = T\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB}$$
$$= \frac{T_{AB}}{1.78 \text{ m}}[-(0.78 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j} + (0 \text{ m})\mathbf{k}]$$
$$\mathbf{T}_{AB} = T_{AB}(-0.4382\mathbf{i} + 0.8989\mathbf{j} + 0\mathbf{k})$$

and

$$\overrightarrow{AC} = (0)\mathbf{i} + (1.6 \text{ m})\mathbf{j} + (1.2 \text{ m})\mathbf{k}$$
$$AC = \sqrt{(0 \text{ m})^2 + (1.6 \text{ m})^2 + (1.2 \text{ m})^2} = 2 \text{ m}$$
$$\mathbf{T}_{AC} = T\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{T_{AC}}{2 \text{ m}}[(0)\mathbf{i} + (1.6 \text{ m})\mathbf{j} + (1.2 \text{ m})\mathbf{k}]$$
$$\mathbf{T}_{AC} = T_{AC}(0.8\mathbf{j} + 0.6\mathbf{k})$$

and

$$\overrightarrow{AD} = (1.3 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j} + (0.4 \text{ m})\mathbf{k}$$
$$AD = \sqrt{(1.3 \text{ m})^2 + (1.6 \text{ m})^2 + (0.4 \text{ m})^2} = 2.1 \text{ m}$$
$$\mathbf{T}_{AD} = T\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{2.1 \text{ m}}[(1.3 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j} + (0.4 \text{ m})\mathbf{k}]$$
$$\mathbf{T}_{AD} = T_{AD}(0.6190\mathbf{i} + 0.7619\mathbf{j} + 0.1905\mathbf{k})$$

PROBLEM 2.121 (Continued)

Finally,

$$\overrightarrow{AE} = -(0.4 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j} - (0.86 \text{ m})\mathbf{k}$$

$$AE = \sqrt{(-0.4 \text{ m})^2 + (1.6 \text{ m})^2 + (-0.86 \text{ m})^2} = 1.86 \text{ m}$$

$$\mathbf{T}_{AE} = T\boldsymbol{\lambda}_{AE} = T_{AE}\frac{\overrightarrow{AE}}{AE}$$

$$= \frac{T_{AE}}{1.86 \text{ m}}[-(0.4 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j} - (0.86 \text{ m})\mathbf{k}]$$

$$\mathbf{T}_{AE} = T_{AE}(-0.2151\mathbf{i} + 0.8602\mathbf{j} - 0.4624\mathbf{k})$$

With the weight of the container $\mathbf{W} = -W\mathbf{j}$, at A we have:

$$\Sigma\mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} - W\mathbf{j} = 0$$

Equating the factors of **i**, **j**, and **k** to zero, we obtain the following linear algebraic equations:

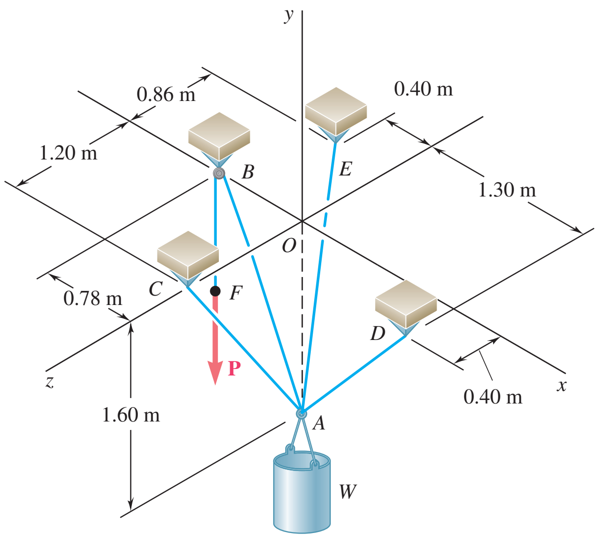
$$-0.4382T_{AB} + 0.6190T_{AD} - 0.2151T_{AE} = 0 \quad (1)$$

$$0.8989T_{AB} + 0.8T_{AC} + 0.7619T_{AD} + 0.8602T_{AE} - W = 0 \quad (2)$$

$$0.6T_{AC} + 0.1905T_{AD} - 0.4624T_{AE} = 0 \quad (3)$$

Knowing that $W = 1000$ N and that because of the pulley system at B $T_{AB} = T_{AD} = P$, where P is the externally applied (unknown) force, we can solve the system of linear Equations (1), (2) and (3) uniquely for P.

$P = 378$ N ◀



PROBLEM 2.122

Knowing that the tension in cable AC of the system described in Problem 2.121 is 150 N, determine (*a*) the magnitude of the force **P**, (*b*) the weight W of the container.

PROBLEM 2.121 A container of weight W is suspended from ring A, to which cables AC and AE are attached. A force **P** is applied to the end F of a third cable that passes over a pulley at B and through ring A and that is attached to a support at D. Knowing that $W = 1000$ N, determine the magnitude of P. (*Hint:* The tension is the same in all portions of cable $FBAD$.)

SOLUTION

Here, as in Problem 2.121, the support of the container consists of the four cables AE, AC, AD, and AB, with the condition that the force in cables AB and AD is equal to the externally applied force P. Thus, with the condition

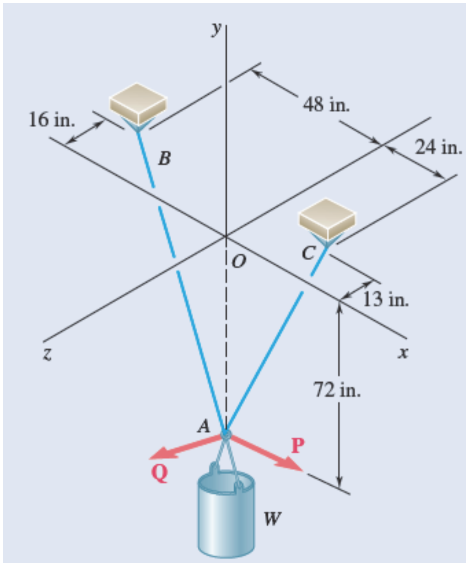
$$T_{AB} = T_{AD} = P$$

and using the linear algebraic equations of Problem 2.131 with $T_{AC} = 150$ N, we obtain

(*a*) $P = 454$ N ◀

(*b*) $W = 1202$ N ◀

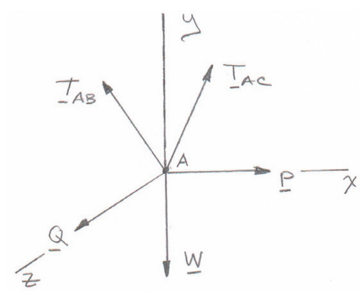
PROBLEM 2.123



A container of weight W is suspended from ring A. Cable BAC passes through the ring and is attached to fixed supports at B and C. Two forces $\mathbf{P} = P\mathbf{i}$ and $\mathbf{Q} = Q\mathbf{k}$ are applied to the ring to maintain the container in the position shown. Knowing that $W = 270$ lb, determine P and Q. (*Hint*: The tension is the same in both portions of cable BAC.)

SOLUTION

Free-Body Diagram of Point *A*:



$$\mathbf{T}_{AB} = T\boldsymbol{\lambda}_{AB}$$
$$= T\frac{\overrightarrow{AB}}{AB}$$
$$= T\frac{(-48\text{ in.})\mathbf{i} + (72\text{ in.})\mathbf{j} - (16\text{ in.})\mathbf{k}}{88\text{ in.}}$$
$$= T\left(-\frac{6}{11}\mathbf{i} + \frac{9}{11}\mathbf{j} - \frac{2}{11}\mathbf{k}\right)$$

$$\mathbf{T}_{AC} = T\boldsymbol{\lambda}_{AC}$$
$$= T\frac{\overrightarrow{AC}}{AC}$$
$$= T\frac{(24\text{ in.})\mathbf{i} + (72\text{ in.})\mathbf{j} + (-13\text{ in.})\mathbf{k}}{77\text{ in.}}$$
$$= T\left(\frac{24}{77}\mathbf{i} + \frac{72}{77}\mathbf{j} - \frac{13}{77}\mathbf{k}\right)$$

$$\Sigma F = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{Q} + \mathbf{P} + \mathbf{W} = 0$$

PROBLEM 2.123 (continued)

Setting coefficients of **i**, **j**, **k** equal to zero:

$$\mathbf{i}: \quad -\frac{6}{11}T + \frac{24}{77}T + P = 0 \qquad -\left(\frac{18}{77}\right)T + P = 0 \qquad (1)$$

$$\mathbf{j}: \quad +\frac{9}{11}T + \frac{72}{77}T - W = 0 \qquad \left(\frac{135}{77}\right)T - W = 0 \qquad (2)$$

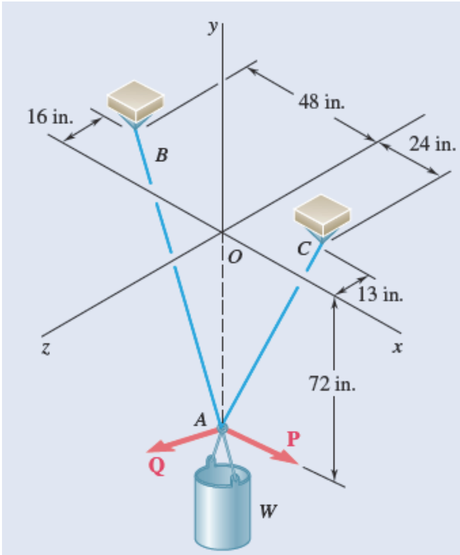
$$\mathbf{k}: \quad -\frac{2}{11}T - \frac{13}{77}T + Q = 0 \qquad -\left(\frac{27}{77}\right)T + Q = 0 \qquad (3)$$

Data: $W = 270 \text{ lb} \quad T = \left(\frac{77}{135}\right)270 \text{ lb} \quad T = 154.0 \text{ lb}$

Substituting for T in Eqn . (1) and (2):

$P = 36.0 \text{ lb}$ ◀

$Q = 54.0 \text{ lb}$ ◀



PROBLEM 2.124

For the system of Prob. 2.123, determine W and P knowing that $Q = 36$ lb.

PROBLEM 2.123 A container of weight W is suspended from ring A. Cable BAC passes through the ring and is attached to fixed supports at B and C. Two forces $\mathbf{P} = P\mathbf{i}$ and $\mathbf{Q} = Q\mathbf{k}$ are applied to the ring to maintain the container in the position shown. Knowing that $W = 270$ lb, determine P and Q. (*Hint*: The tension is the same in both portions of cable BAC.)

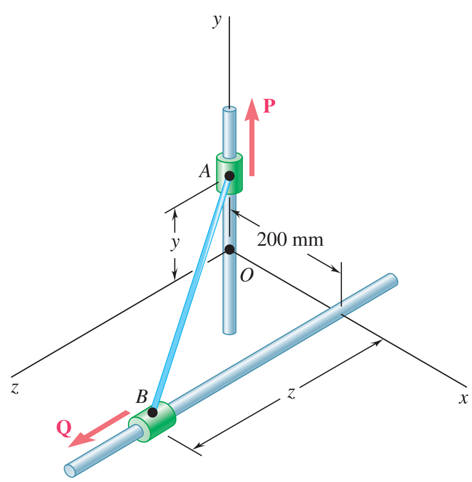
SOLUTION

Refer to Problem 2.123 for the figure and analysis resulting in Equations (1), (2), and (3). Setting $Q = 36$ lb we have:

Eq. (3): $$T = \left(\frac{77}{27}\right) 36 \text{ lb} \qquad T = 102.667 \text{ lb}$$

Eq. (1): $$-\frac{18}{77}(102.667 \text{ lb}) + \text{P} = 0 \qquad P = 24.0 \text{ lb} \blacktriangleleft$$

Eq. (2): $$\frac{135}{77}(102.667 \text{ lb}) - \text{W} = 0 \qquad W = 180.0 \text{ lb} \blacktriangleleft$$



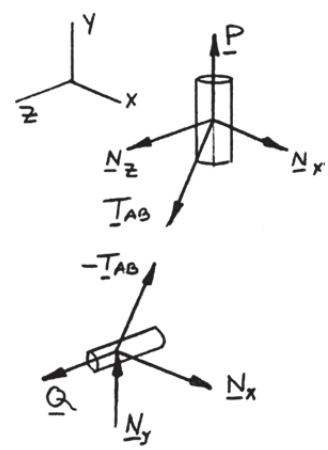
PROBLEM 2.125

Collars A and B are connected by a 525-mm-long wire and can slide freely on frictionless rods. If a force $\mathbf{P} = (341\text{ N})\mathbf{j}$ is applied to collar A, determine (a) the tension in the wire when $y = 155$ mm, (b) the magnitude of the force $\mathbf{Q}$ required to maintain the equilibrium of the system.

SOLUTION

For both Problems 2.125 and 2.126:

Free-Body Diagrams of Collars:



$$(AB)^2 = x^2 + y^2 + z^2$$

Here $$(0.525\text{ m})^2 = (0.20\text{ m})^2 + y^2 + z^2$$

or $$y^2 + z^2 = 0.23563\text{ m}^2$$

Thus, when y given, z is determined,

Now
$$\boldsymbol{\lambda}_{AB} = \frac{\overrightarrow{AB}}{AB}$$
$$= \frac{1}{0.525\text{ m}}(0.20\mathbf{i} - y\mathbf{j} + z\mathbf{k})\text{m}$$
$$= 0.38095\mathbf{i} - 1.90476y\mathbf{j} + 1.90476z\mathbf{k}$$

Where y and z are in units of meters, m.

From the F.B. Diagram of collar A: $\Sigma\mathbf{F} = 0{:}\quad N_x\mathbf{i} + N_z\mathbf{k} + P\mathbf{j} + T_{AB}\lambda_{AB} = 0$

Setting the $\mathbf{j}$ coefficient to zero gives $P - (1.90476y)T_{AB} = 0$

With $$P = 341\text{ N}$$

$$T_{AB} = \frac{341\text{ N}}{1.90476y}$$

Now, from the free body diagram of collar B: $\Sigma\mathbf{F} = 0{:}\quad N_x\mathbf{i} + N_y\mathbf{j} + Q\mathbf{k} - T_{AB}\boldsymbol{\lambda}_{AB} = 0$

Setting the $\mathbf{k}$ coefficient to zero gives $Q - T_{AB}(1.90476z) = 0$

And using the above result for T_{AB}, we have $$Q = T_{AB}z = \frac{341\text{ N}}{(1.90476)y}(1.90476z) = \frac{(341\text{ N})(z)}{y}$$

PROBLEM 2.125 (Continued)

Then from the specifications of the problem, $y = 155 \text{ mm} = 0.155 \text{ m}$

$$z^2 = 0.23563 \text{ m}^2 - (0.155 \text{ m})^2$$
$$z = 0.46 \text{ m}$$

and

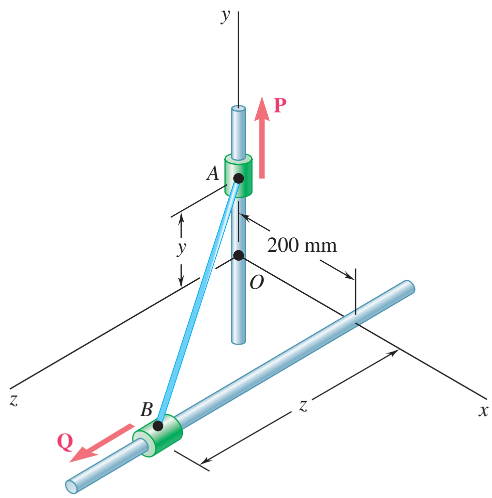
(*a*)
$$T_{AB} = \frac{341 \text{ N}}{0.155(1.90476)}$$
$$= 1155.00 \text{ N}$$

or $T_{AB} = 1155 \text{ N}$ ◀

and

(*b*)
$$Q = \frac{341 \text{ N}(0.46 \text{ m})(0.866)}{(0.155 \text{ m})}$$
$$= (1012.00 \text{ N})$$

or $Q = 1012 \text{ N}$ ◀



PROBLEM 2.126

Solve Problem 2.125 assuming that $y = 275$ mm.

PROBLEM 2.125 Collars A and B are connected by a 525-mm-long wire and can slide freely on frictionless rods. If a force $\mathbf{P} = (341\text{ N})\mathbf{j}$ is applied to collar A, determine (a) the tension in the wire when $y = 155$ mm, (b) the magnitude of the force $\mathbf{Q}$ required to maintain the equilibrium of the system.

SOLUTION

From the analysis of Problem 2.125, particularly the results:

$$y^2 + z^2 = 0.23563\text{ m}^2$$

$$T_{AB} = \frac{341\text{ N}}{1.90476y}$$

$$Q = \frac{341\text{ N}}{y}z$$

With $y = 275\text{ mm} = 0.275\text{ m}$, we obtain:

$$z^2 = 0.23563\text{ m}^2 - (0.275\text{ m})^2$$

$$z = 0.40\text{ m}$$

and

(a)

$$T_{AB} = \frac{341\text{ N}}{(1.90476)(0.275\text{ m})} = 651.00$$

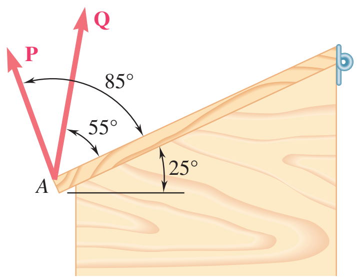
or $T_{AB} = 651\text{ N}$ ◀

and

(b)

$$Q = \frac{341\text{ N}(0.40\text{ m})}{(0.275\text{ m})}$$

or $Q = 496\text{ N}$ ◀

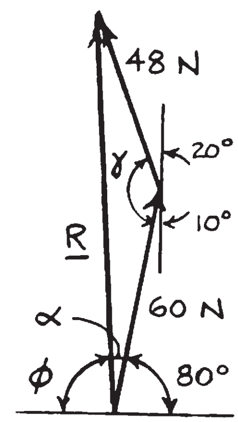


PROBLEM 2.127

Two forces **P** and **Q** are applied to the lid of a storage bin as shown. Knowing that $P = 48$ N and $Q = 60$ N, determine by trigonometry the magnitude and direction of the resultant of the two forces.

SOLUTION

Using the force triangle and the laws of cosines and sines:



We have

$$\gamma = 180° - (20° + 10°) = 150°$$

Then

$$R^2 = (48\text{ N})^2 + (60\text{ N})^2 - 2(48\text{ N})(60\text{ N})\cos 150°$$

$$R = 104.366\text{ N}$$

and

$$\frac{48\text{ N}}{\sin\alpha} = \frac{104.366\text{ N}}{\sin 150°}$$

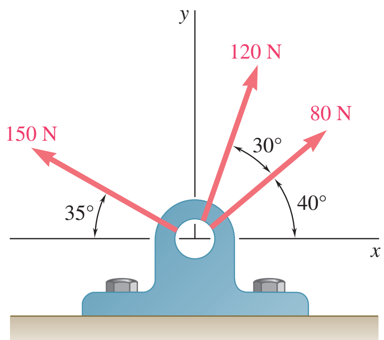
$$\sin\alpha = 0.22996$$

$$\alpha = 13.2947°$$

Hence:

$$\phi = 180° - \alpha - 80° = 180° - 13.2947° - 80° = 86.705°$$

R = 104.4 N ⦣ 86.7° ◀



PROBLEM 2.128

Determine the *x* and *y* components of each of the forces shown.

SOLUTION

80-N Force:

$$F_x = +(80\text{ N})\cos 40° \qquad F_x = 61.3\text{ N} \blacktriangleleft$$

$$F_y = +(80\text{ N})\sin 40° \qquad F_y = 51.4\text{ N} \blacktriangleleft$$

120-N Force:

$$F_x = +(120\text{ N})\cos 70° \qquad F_x = 41.0\text{ N} \blacktriangleleft$$

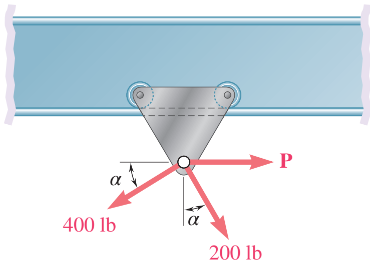
$$F_y = +(120\text{ N})\sin 70° \qquad F_y = 112.8\text{ N} \blacktriangleleft$$

150-N Force:

$$F_x = -(150\text{ N})\cos 35° \qquad F_x = -122.9\text{ N} \blacktriangleleft$$

$$F_y = +(150\text{ N})\sin 35° \qquad F_y = 86.0\text{ N} \blacktriangleleft$$

PROBLEM 2.129



A hoist trolley is subjected to the three forces shown. Knowing that $\alpha = 40°$, determine (*a*) the required magnitude of the force **P** if the resultant of the three forces is to be vertical, (*b*) the corresponding magnitude of the resultant.

SOLUTION

$$R_x = \xrightarrow{+} \Sigma F_x = P + (200 \text{ lb})\sin 40° - (400 \text{ lb})\cos 40°$$

$$R_x = P - 177.860 \text{ lb} \qquad (1)$$

$$R_y = +\downarrow \Sigma F_y = (200 \text{ lb})\cos 40° + (400 \text{ lb})\sin 40°$$

$$R_y = 410.32 \text{ lb} \qquad (2)$$

(*a*) For **R** to be vertical, we must have $R_x = 0$.

Set $R_x = 0$ in Eq. (1)

$$0 = P - 177.860 \text{ lb}$$

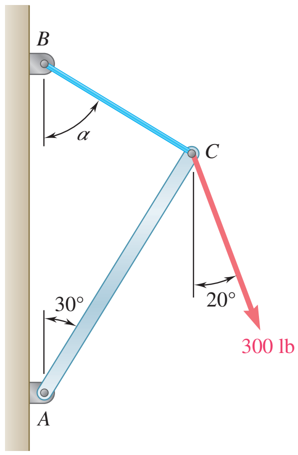
$$P = 177.860 \text{ lb}$$

$P = 177.9$ lb ◀

(*b*) Since **R** is to be vertical:

$$R = R_y = 410 \text{ lb}$$

$R = 410$ lb ◀

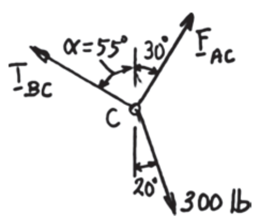


PROBLEM 2.130

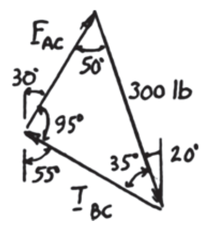
Knowing that $\alpha = 55°$ and that boom AC exerts on pin C a force directed along line AC, determine (*a*) the magnitude of that force, (*b*) the tension in cable BC.

SOLUTION

Free-Body Diagram



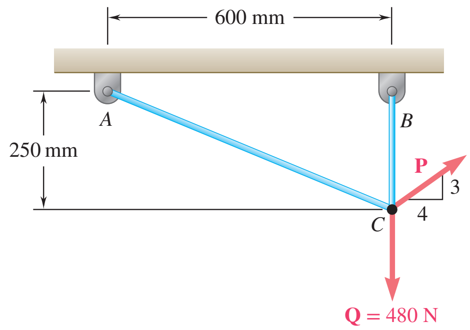
Force Triangle



Law of sines:
$$\frac{F_{AC}}{\sin 35°} = \frac{T_{BC}}{\sin 50°} = \frac{300 \text{ lb}}{\sin 95°}$$

(*a*)
$$F_{AC} = \frac{300 \text{ lb}}{\sin 95°} \sin 35°$$
$F_{AC} = 172.7$ lb ◀

(*b*)
$$T_{BC} = \frac{300 \text{ lb}}{\sin 95°} \sin 50°$$
$T_{BC} = 231$ lb ◀

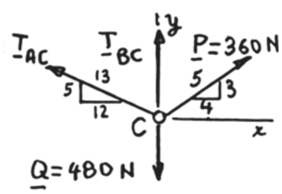


PROBLEM 2.131

Two cables are tied together at C and loaded as shown. Knowing that $P = 360$ N, determine the tension (*a*) in cable AC, (*b*) in cable BC.

SOLUTION

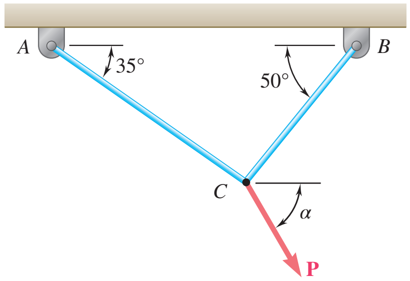
Free Body: *C*



(*a*) $\Sigma F_x = 0: \quad -\frac{12}{13}T_{AC} + \frac{4}{5}(360\text{ N}) = 0$ $\qquad T_{AC} = 312\text{ N}$ ◀

(*b*) $\Sigma F_y = 0: \quad \frac{5}{13}(312\text{ N}) + T_{BC} + \frac{3}{5}(360\text{ N}) - 480\text{ N} = 0$

$T_{BC} = 480\text{ N} - 120\text{ N} - 216\text{ N}$ $\qquad T_{BC} = 144.0\text{ N}$ ◀

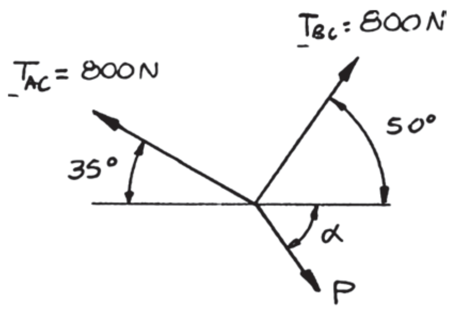


PROBLEM 2.132

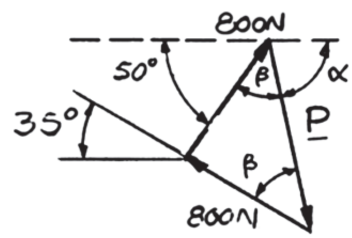
Two cables tied together at *C* are loaded as shown. Knowing that the maximum allowable tension in each cable is 800 N, determine (*a*) the magnitude of the largest force **P** that can be applied at *C*, (*b*) the corresponding value of α.

SOLUTION

Free-Body Diagram: *C*



Force Triangle



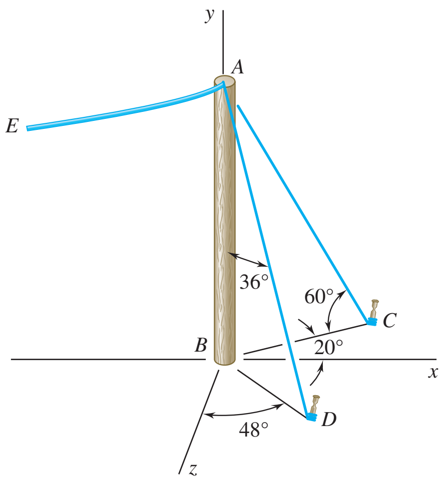
Force triangle is isosceles with

$$2\beta = 180° - 85°$$
$$\beta = 47.5°$$

(*a*) $$P = 2(800\text{ N})\cos 47.5° = 1081\text{ N}$$

Since $P > 0$, the solution is correct. $P = 1081\text{ N}$ ◀

(*b*) $$\alpha = 180° - 50° - 47.5° = 82.5°$$ $\alpha = 82.5°$ ◀



PROBLEM 2.133

The end of the coaxial cable *AE* is attached to the pole *AB*, which is strengthened by the guy wires *AC* and *AD*. Knowing that the tension in wire *AC* is 120 lb, determine (*a*) the components of the force exerted by this wire on the pole, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

SOLUTION

(*a*)

$$F_x = (120 \text{ lb})\cos 60^\circ \cos 20^\circ$$

$$F_x = 56.382 \text{ lb} \qquad F_x = +56.4 \text{ lb} \blacktriangleleft$$

$$F_y = -(120 \text{ lb})\sin 60^\circ$$

$$F_y = -103.923 \text{ lb} \qquad F_y = -103.9 \text{ lb} \blacktriangleleft$$

$$F_z = -(120 \text{ lb})\cos 60^\circ \sin 20^\circ$$

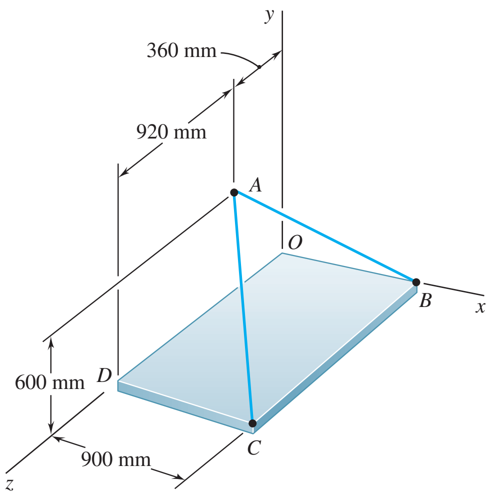
$$F_z = -20.521 \text{ lb} \qquad F_z = -20.5 \text{ lb} \blacktriangleleft$$

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{56.382 \text{ lb}}{120 \text{ lb}} \qquad \theta_x = 62.0^\circ \blacktriangleleft$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{-103.923 \text{ lb}}{120 \text{ lb}} \qquad \theta_y = 150.0^\circ \blacktriangleleft$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{-20.52 \text{ lb}}{120 \text{ lb}} \qquad \theta_z = 99.8^\circ \blacktriangleleft$$



PROBLEM 2.134

Knowing that the tension in cable *AC* is 2130 N, determine the components of the force exerted on the plate at *C*.

SOLUTION

$$\overrightarrow{CA} = -(900 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} - (920 \text{ mm})\mathbf{k}$$

$$CA = \sqrt{(900 \text{ mm})^2 + (600 \text{ mm})^2 + (920 \text{ mm})^2}$$

$$= 1420 \text{ mm}$$

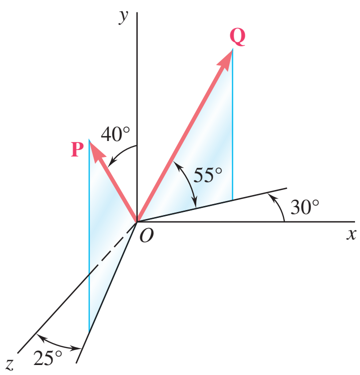
$$\mathbf{T}_{CA} = T_{CA}\boldsymbol{\lambda}_{CA}$$

$$= T_{CA}\frac{\overrightarrow{CA}}{CA}$$

$$\mathbf{T}_{CA} = \frac{2130 \text{ N}}{1420 \text{ mm}}[-(900 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} - (920 \text{ mm})\mathbf{k}]$$

$$= -(1350 \text{ N})\mathbf{i} + (900 \text{ N})\mathbf{j} - (1380 \text{ N})\mathbf{k}$$

$$(T_{CA})_x = -1350 \text{ N}, \quad (T_{CA})_y = 900 \text{ N}, \quad (T_{CA})_z = -1380 \text{ N} \blacktriangleleft$$



PROBLEM 2.135

Find the magnitude and direction of the resultant of the two forces shown knowing that $P = 600$ N and $Q = 450$ N.

SOLUTION

$$\mathbf{P} = (600\text{ N})[\sin 40°\sin 25°\mathbf{i} + \cos 40°\mathbf{j} + \sin 40°\cos 25°\mathbf{k}]$$
$$= (162.992\text{ N})\mathbf{i} + (459.63\text{ N})\mathbf{j} + (349.54\text{ N})\mathbf{k}$$
$$\mathbf{Q} = (450\text{ N})[\cos 55°\cos 30°\mathbf{i} + \sin 55°\mathbf{j} - \cos 55°\sin 30°\mathbf{k}]$$
$$= (223.53\text{ N})\mathbf{i} + (368.62\text{ N})\mathbf{j} - (129.055\text{ N})\mathbf{k}$$
$$\mathbf{R} = \mathbf{P} + \mathbf{Q}$$
$$= (386.52\text{ N})\mathbf{i} + (828.25\text{ N})\mathbf{j} + (220.49\text{ N})\mathbf{k}$$
$$R = \sqrt{(386.52\text{ N})^2 + (828.25\text{ N})^2 + (220.49\text{ N})^2}$$
$$= 940.22\text{ N}$$

$R = 940$ N ◀

$$\cos\theta_x = \frac{R_x}{R} = \frac{386.52\text{ N}}{940.22\text{ N}}$$

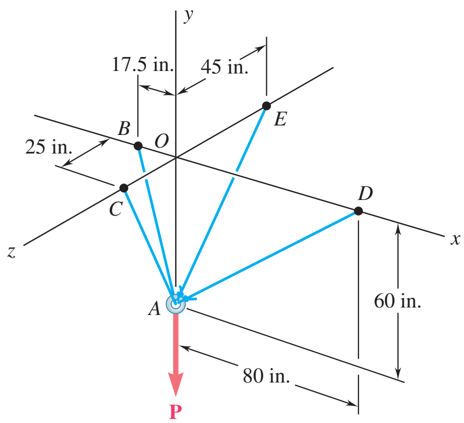
$\theta_x = 65.7°$ ◀

$$\cos\theta_y = \frac{R_y}{R} = \frac{828.25\text{ N}}{940.22\text{ N}}$$

$\theta_y = 28.2°$ ◀

$$\cos\theta_z = \frac{R_z}{R} = \frac{220.49\text{ N}}{940.22\text{ N}}$$

$\theta_z = 76.4°$ ◀

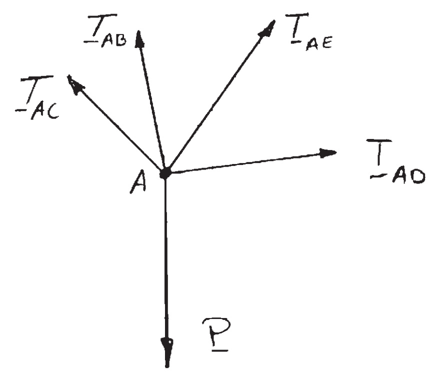


PROBLEM 2.136

Cable *BAC* passes through a frictionless ring *A* and is attached to fixed supports at *B* and *C*, while cables *AD* and *AE* are both tied to the ring and are attached, respectively, to supports at *D* and *E*. Knowing that a 200-lb vertical load **P** is applied to ring *A*, determine the tension in each of the three cables.

SOLUTION

Free Body Diagram at *A*:



Since T_{BAC} = tension in cable *BAC*, it follows that

$$T_{AB} = T_{AC} = T_{BAC}$$

$$\mathbf{T}_{AB} = T_{BAC}\boldsymbol{\lambda}_{AB} = T_{BAC}\frac{(-17.5\text{ in.})\mathbf{i} + (60\text{ in.})\mathbf{j}}{62.5\text{ in.}} = T_{BAC}\left(\frac{-17.5}{62.5}\mathbf{i} + \frac{60}{62.5}\mathbf{j}\right)$$

$$\mathbf{T}_{AC} = T_{BAC}\boldsymbol{\lambda}_{AC} = T_{BAC}\frac{(60\text{ in.})\mathbf{i} + (25\text{ in.})\mathbf{k}}{65\text{ in.}} = T_{BAC}\left(\frac{60}{65}\mathbf{j} + \frac{25}{65}\mathbf{k}\right)$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{(80\text{ in.})\mathbf{i} + (60\text{ in.})\mathbf{j}}{100\text{ in.}} = T_{AD}\left(\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{j}\right)$$

$$\mathbf{T}_{AE} = T_{AE}\boldsymbol{\lambda}_{AE} = T_{AE}\frac{(60\text{ in.})\mathbf{j} - (45\text{ in.})\mathbf{k}}{75\text{ in.}} = T_{AE}\left(\frac{4}{5}\mathbf{j} - \frac{3}{5}\mathbf{k}\right)$$

PROBLEM 2.136 (Continued)

Substituting into $\Sigma \mathbf{F}_A = 0$, setting $\mathbf{P} = (-200 \text{ lb})\mathbf{j}$, and setting the coefficients of **i**, **j**, **k** equal to ϕ, we obtain the following three equilibrium equations:

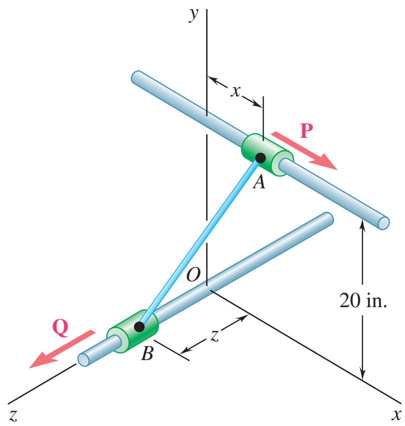
From **i**: $$-\frac{17.5}{62.5}T_{BAC} + \frac{4}{5}T_{AD} = 0 \quad (1)$$

From **j**: $$\left(\frac{60}{62.5} + \frac{60}{65}\right)T_{BAC} + \frac{3}{5}T_{AD} + \frac{4}{5}T_{AE} - 200 \text{ lb} = 0 \quad (2)$$

From **k**: $$\frac{25}{65}T_{BAC} - \frac{3}{5}T_{AE} = 0 \quad (3)$$

Solving the system of linear equations using conventional algorithms gives:

$$T_{BAC} = 76.7 \text{ lb};\ T_{AD} = 26.9 \text{ lb};\ T_{AE} = 49.2 \text{ lb} \blacktriangleleft$$



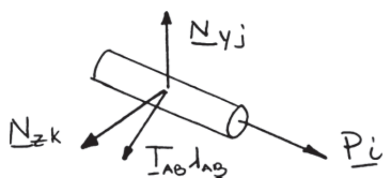
PROBLEM 2.137

Collars A and B are connected by a 25-in.-long wire and can slide freely on frictionless rods. If a 60-lb force **Q** is applied to collar B as shown, determine (*a*) the tension in the wire when $x = 9$ in., (*b*) the corresponding magnitude of the force **P** required to maintain the equilibrium of the system.

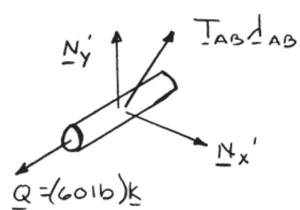
SOLUTION

Free-Body Diagrams of Collars:

***A*:**



***B*:**



$$\boldsymbol{\lambda}_{AB} = \frac{\overrightarrow{AB}}{AB} = \frac{-x\mathbf{i} - (20 \text{ in.})\mathbf{j} + z\mathbf{k}}{25 \text{ in.}}$$

Collar A: $\Sigma \mathbf{F} = 0$: $P\mathbf{i} + N_y\mathbf{j} + N_z\mathbf{k} + T_{AB}\boldsymbol{\lambda}_{AB} = 0$

Substitute for $\boldsymbol{\lambda}_{AB}$ and set coefficient of **i** equal to zero:

$$P - \frac{T_{AB}x}{25 \text{ in.}} = 0 \qquad (1)$$

Collar B: $\Sigma \mathbf{F} = 0$: $(60 \text{ lb})\mathbf{k} + N'_x\mathbf{i} + N'_y\mathbf{j} - T_{AB}\boldsymbol{\lambda}_{AB} = 0$

Substitute for $\boldsymbol{\lambda}_{AB}$ and set coefficient of **k** equal to zero:

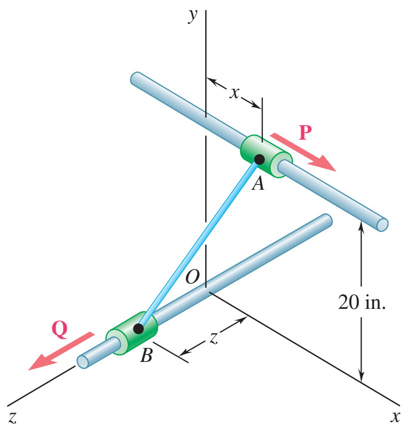
$$60 \text{ lb} - \frac{T_{AB}z}{25 \text{ in.}} = 0 \qquad (2)$$

(*a*) $x = 9$ in. $\quad (9 \text{ in.})^2 + (20 \text{ in.})^2 + z^2 = (25 \text{ in.})^2$

$$z = 12 \text{ in.}$$

From Eq. (2): $\dfrac{60 \text{ lb} - T_{AB}(12 \text{ in.})}{25 \text{ in.}}$ $\quad T_{AB} = 125.0 \text{ lb}$ ◀

(*b*) From Eq. (1): $P = \dfrac{(125.0 \text{ lb})(9 \text{ in.})}{25 \text{ in.}}$ $\quad P = 45.0 \text{ lb}$ ◀



PROBLEM 2.138

Collars A and B are connected by a 25-in.-long wire and can slide freely on frictionless rods. Determine the distances x and z for which the equilibrium of the system is maintained when $P = 120$ lb and $Q = 60$ lb.

SOLUTION

See Problem 2.137 for the diagrams and analysis leading to Equations (1) and (2) below:

$$P = \frac{T_{AB}x}{25 \text{ in.}} = 0 \qquad (1)$$

$$60 \text{ lb} - \frac{T_{AB}z}{25 \text{ in.}} = 0 \qquad (2)$$

For $P = 120$ lb, Eq. (1) yields
$$T_{AB}x = (25 \text{ in.})(20 \text{ lb}) \qquad (1')$$

From Eq. (2):
$$T_{AB}z = (25 \text{ in.})(60 \text{ lb}) \qquad (2')$$

Dividing Eq. (1′) by (2′),
$$\frac{x}{z} = 2 \qquad (3)$$

Now write
$$x^2 + z^2 + (20 \text{ in.})^2 = (25 \text{ in.})^2 \qquad (4)$$

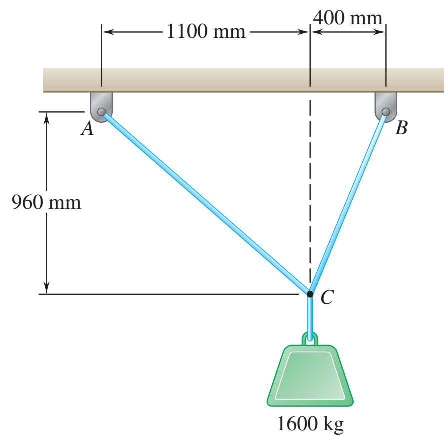
Solving (3) and (4) simultaneously,

$$4z^2 + z^2 + 400 = 625$$
$$z^2 = 45$$
$$z = 6.7082 \text{ in.}$$

From Eq. (3):
$$x = 2z = 2(6.7082 \text{ in.})$$
$$= 13.4164 \text{ in.}$$

$$x = 13.42 \text{ in.}, \quad z = 6.71 \text{ in.}$$ ◀

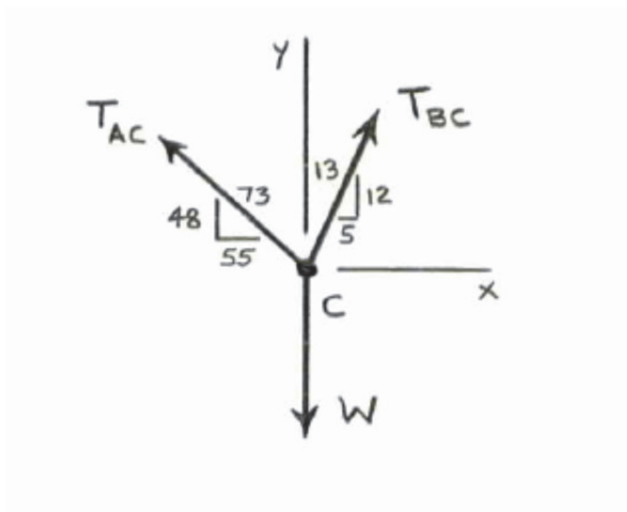
PROBLEM 2F1



Two cables are tied together at C and loaded as shown. Draw the free-body diagram needed to determine the tension in AC and BC.

SOLUTION

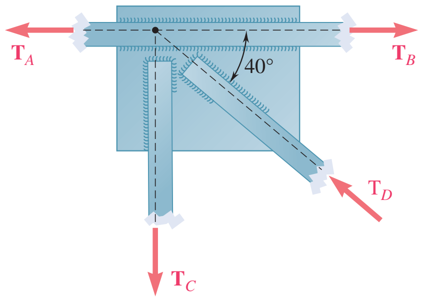
Free-Body Diagram of Point *C*:



$$W = (1600\text{ kg})(9.81\text{ m/s}^2)$$

$$W = 15.6960(10^3)\text{ N}$$

$$W = 15.696\text{ kN}$$

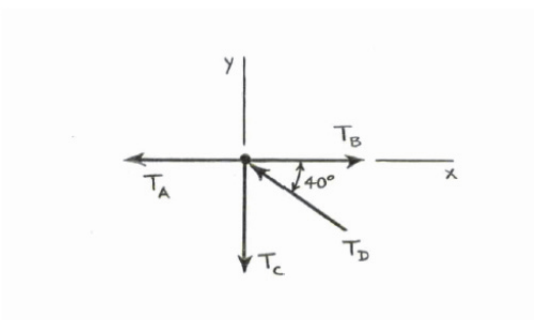


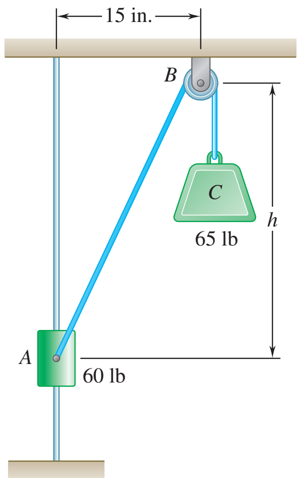
PROBLEM 2.F2

Two forces of magnitude $T_A = 8$ kips and $T_B = 15$ kips are applied as shown to a welded connection. Knowing that the connection is in equilibrium, draw the free-body diagram needed to determine the magnitudes of the forces T_C and T_D.

SOLUTION

Free-Body Diagram of Point *E*:



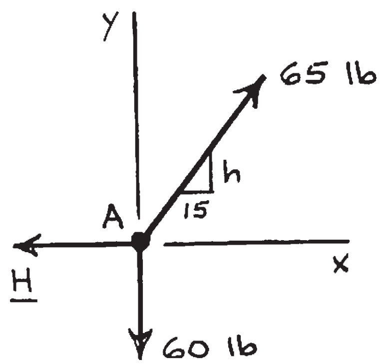


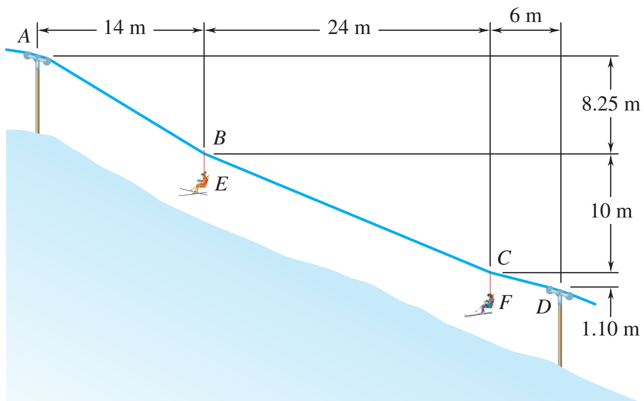
PROBLEM 2.F3

The 60-lb collar A can slide on a frictionless vertical rod and is connected as shown to a 65-lb counterweight C. Draw the free-body diagram needed to determine the value of h for which the system is in equilibrium.

SOLUTION

Free-Body Diagram of Point *A*:



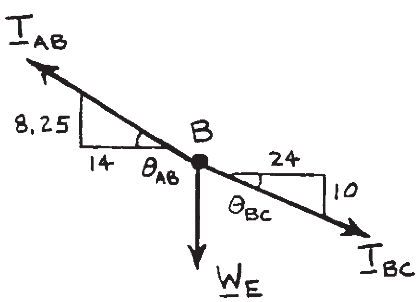


PROBLEM 2.F4

A chairlift has been stopped in the position shown. Knowing that each chair weighs 250 N and that the skier in chair E weighs 765 N, draw the free-body diagrams needed to determine the weight of the skier in chair F.

SOLUTION

Free-Body Diagram of Point *B*:



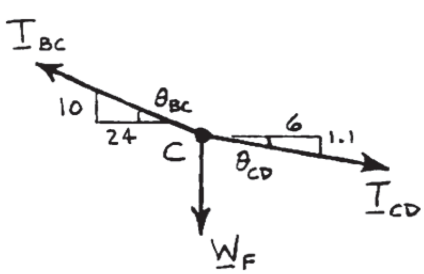
$$W_E = 250\text{ N} + 765\text{ N} = 1015\text{ N}$$

$$\theta_{AB} = \tan^{-1}\frac{8.25}{14} = 30.510°$$

$$\theta_{BC} = \tan^{-1}\frac{10}{24} = 22.620°$$

Use this free body to determine T_{AB} and T_{BC}.

Free-Body Diagram of Point *C*:

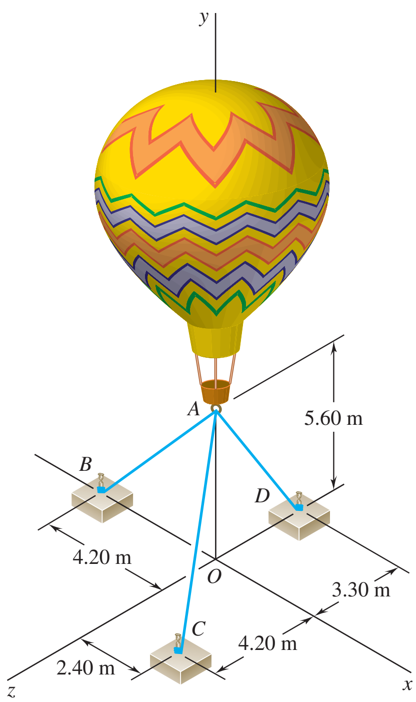


$$\theta_{CD} = \tan^{-1}\frac{1.1}{6} = 10.3889°$$

Use this free body to determine T_{CD} and W_F.

Then weight of skier W_S is found by

$$W_S = W_F - 250\text{ N} \blacktriangleleft$$

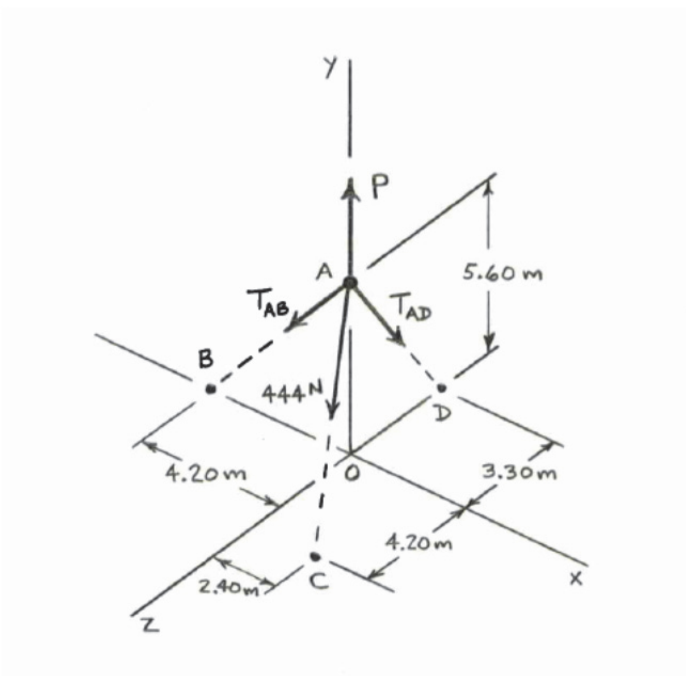


PROBLEM 2.F5

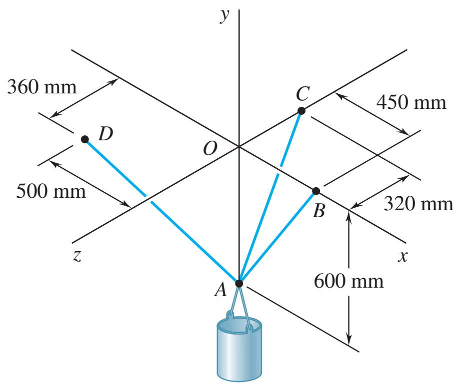
Three cables are used to tether a balloon as shown. Knowing that the tension in cable *AC* is 444 N, draw the free-body diagram needed to determine the vertical force **P** exerted by the balloon at *A*.

SOLUTION

Free-Body Diagram of Point *A*:



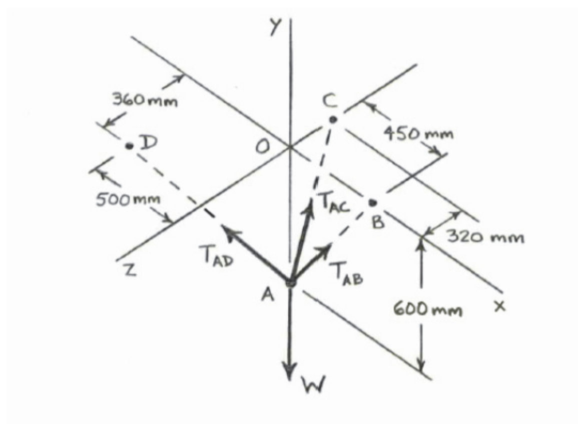
PROBLEM 2.F6



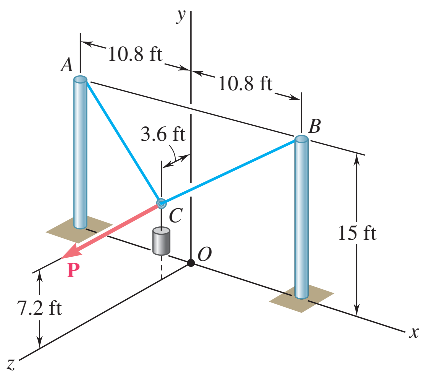
A container of mass $m = 120$ kg is supported by three cables as shown. Draw the free-body diagram needed to determine the tension in each cable.

SOLUTION

Free-Body Diagram of Point *A*:



$$W = (120 \text{ kg})(9.81 \text{ m/s}^2)$$
$$= 1177.2 \text{ N}$$

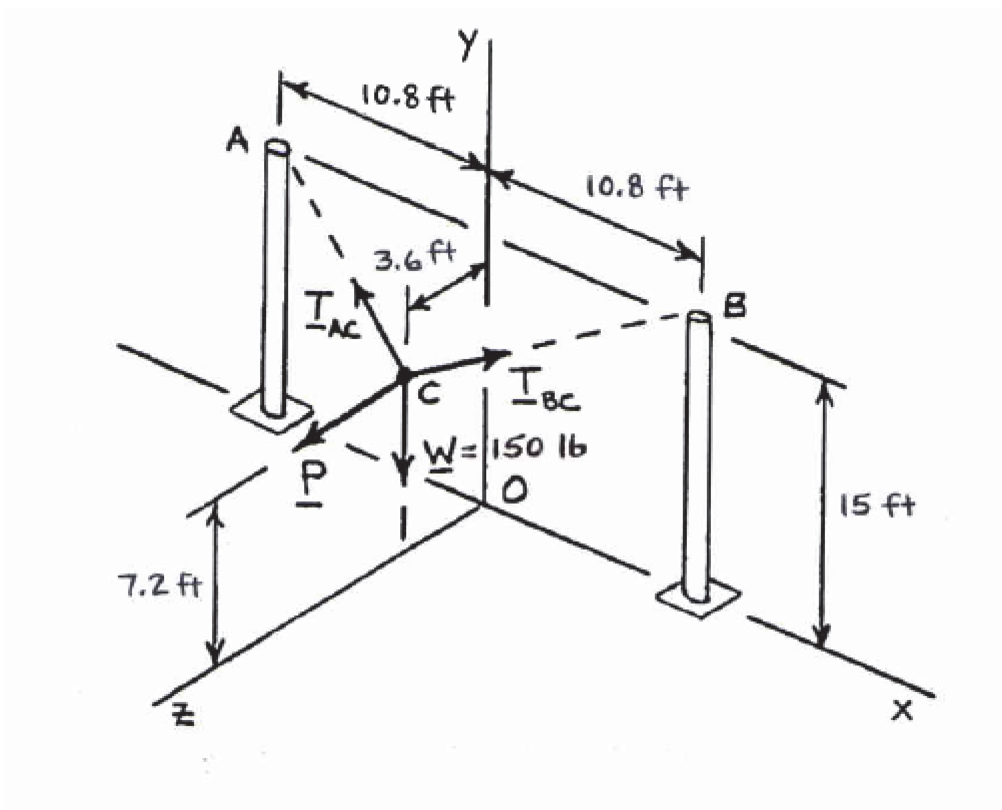


PROBLEM 2.F7

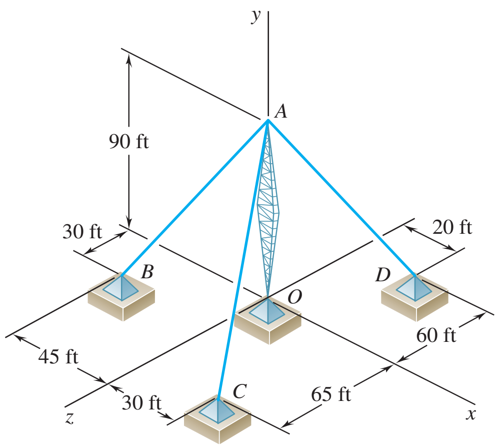
A 150-lb cylinder is supported by two cables *AC* and *BC* that are attached to the top of vertical posts. A horizontal force **P**, which is perpendicular to the plane containing the posts, holds the cylinder in the position shown. Draw the free-body diagram needed to determine the magnitude of **P** and the force in each cable.

SOLUTION

Free-Body Diagram of Point *C*:



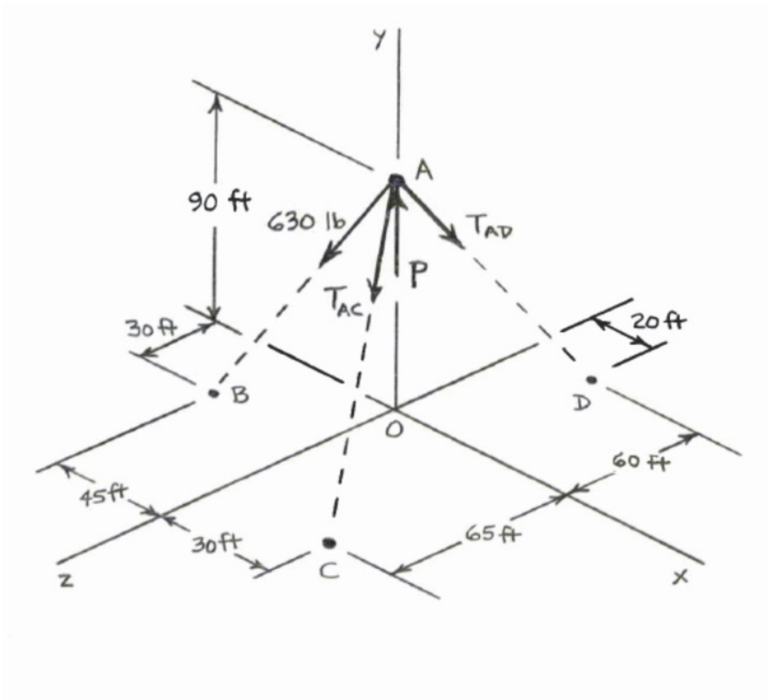
PROBLEM 2.F8



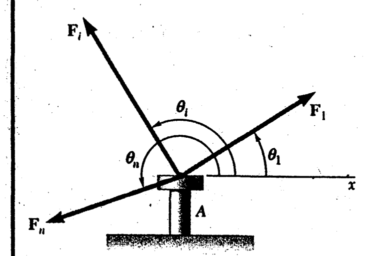
A transmission tower is held by three guy wires attached to a pin at *A* and anchored by bolts at *B*, *C*, and *D*. Knowing that the tension in wire *AB* is 630 lb, draw the free-body diagram needed to determine the vertical force **P** exerted by the tower on the pin at *A*.

SOLUTION

Free-Body Diagram of point *A*:



2.C1



GIVEN:

$F_1, F_2, \ldots, F_n$ AND $\theta_1, \theta_2, \ldots \theta_n$

FIND:

MAGNITUDE AND DIRECTION OF RESULTANT, USING DATA OF PROBS. 2.32, 2.33, 2.35, AND 2.38.

ANALYSIS

$$R_x = \sum_{i=1}^{n} F_i \cos\theta_i, \quad R_y = \sum_{i=1}^{n} F_i \sin\theta \quad (1)$$

WE HAVE

$$R = \sqrt{R_x^2 + R_y^2} \quad (2)$$

AND

$$\theta_R = \tan^{-1}\frac{R_y}{R_x} \quad (3)$$

LET θ_R^* BE THE VALUE DEFINED BY EQ. (3) AND SUCH THAT $-90° \le \theta_R^* \le +90°$. THEN, THE CORRECT ANSWER WILL BE

IF $R_x \ge 0$ AND $R_y \ge 0$: $\quad \theta_R = \theta_R^* \quad (4)$

IF $R_x \ge 0$ AND $R_y < 0$: $\quad \theta_R = 360° + \theta_R^* \quad (4')$

IF $R_x < 0$: $\quad \theta_R = 180° + \theta_R^* \quad (4'')$

OUTLINE OF PROGRAM

ENTER PROBLEM NUMBER AND NUMBER OF FORCES n.
COMPUTE R_x, R_y, R, AND θ_R^* FROM EQS. ABOVE
DETERMINE θ_R FROM EQ. (4), (4'), OR (4")

PROGRAM OUTPUT

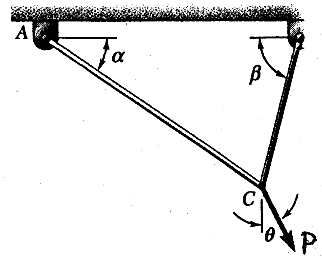
```
Data of Prob. 2.32
Number of forces : N = 3
Force and angle (F, TH)? 80,40
Force and angle (F, TH)? 120,70
Force and angle (F, TH)? 150,145
Resultant =   251.065
Angle between resultant and x axis =  94.69 degrees

Data of Prob. 2.33
Number of forces : N = 3
Force and angle (F, TH)? 60,25
Force and angle (F, TH)? 50,220
Force and angle (F, TH)? 40,300
Resultant =    54.931
Angle between resultant and x axis = 311.05 degrees

Data of Prob. 2.35
Number of forces : N = 3
Force and angle (F, TH)? 200,215
Force and angle (F, TH)? 150,295
Force and angle (F, TH)? 100,325
Resultant =   308.576
Angle between resultant and x axis = 266.56 degrees

Data of Prob. 2.38
Number of forces : N = 3
Force and angle (F, TH)? 60,20
Force and angle (F, TH)? 80,95
Force and angle (F, TH)? 120,5
Resultant =   201.975
Angle between resultant and x axis =  33.23 degrees
```

2.C2



GIVEN:

LOAD P SUPPORTED BY TWO CABLES AS SHOWN

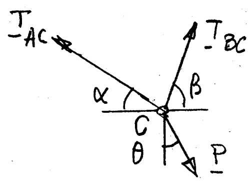
FIND FOR EACH SET OF VALUES SHOWN AND FOR VALUES OF θ FROM $\theta_1 = \beta - 90°$ TO $\theta_2 = 90° - \alpha$ USING INCREMENTS $\Delta\theta$:

(a) TENSION IN EACH CABLE
(b) VALUE OF θ FOR WHICH THE TENSION IN THE CABLES IS AS SMALL AS POSSIBLE.
(c) CORRESPONDING TENSION

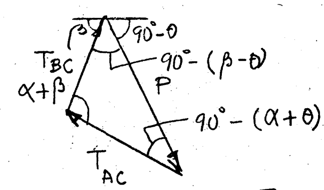
(1) $\alpha = 35°$, $\beta = 75°$, $P = 400$ lb, $\Delta\theta = 5°$
(2) $\alpha = 50°$, $\beta = 30°$, $P = 600$ lb, $\Delta\theta = 10°$
(3) $\alpha = 40°$, $\beta = 60°$, $P = 250$ lb, $\Delta\theta = 5°$

ANALYSIS

F.B. DIAGRAM OF C



FORCE TRIANGLE



LAW OF SINES: $\dfrac{P}{\sin(\alpha+\beta)} = \dfrac{T_{AC}}{\cos(\beta-\theta)} = \dfrac{T_{BC}}{\cos(\alpha+\theta)}$

$$T_{AC} = P\frac{\cos(\beta-\theta)}{\sin(\alpha+\beta)} \qquad T_{BC} = P\frac{\cos(\alpha+\theta)}{\sin(\alpha+\beta)} \quad (1)$$

TENSION IN EACH CABLE IS AS SMALL AS POSSIBLE WHEN $T_{AC} = T_{BC}$, THAT IS, WHEN $\beta - \theta = \alpha + \theta$, $\theta = \frac{\beta - \alpha}{2}$

OUTLINE OF PROGRAM

ENTER SET NUMBER AND VALUES OF $\alpha, \beta, P, \Delta\theta$
COMPUTE $\theta_1 = \beta - 90°$ AND $\theta_2 = 90° - \alpha$
FOR VALUES OF θ FROM θ_1 TO θ_2, USING INCREMENTS $\Delta\theta$, COMPUTE T_{AC} AND T_{BC} FROM EQS. (1)
PRINT θ, T_{AC}, T_{BC}
CHECK THAT T_{AC} AND T_{BC} ARE EQUAL AND MINIMUM FOR $\theta = \frac{1}{2}(\beta - \alpha)$.

PROGRAM OUTPUT

```
Set No.1
Angle ALPHA = 35
Angle BETA = 75
Magnitude of load P = 400
Increment of THETA = 5

  THETA        TAB       TAC

 -15.000     -0.000   400.000
 -10.000     37.100   385.789
  -5.000     73.917   368.642
   0.000    110.172   348.689
   5.000    145.588   326.083
  10.000    179.896   300.995
  15.000    212.836   273.616
  20.000    244.155   244.155
  25.000    273.616   212.836
  30.000    300.995   179.896
  35.000    326.083   145.588
  40.000    348.689   110.172
  45.000    368.642    73.917
  50.000    385.789    37.100
  55.000    400.000    -0.000
```

(b) ▶ at 20.000 row ◀ (c)

continued

2.C2 continued

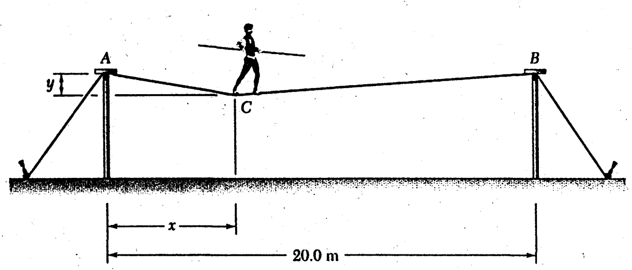
Set No.2
Angle ALPHA = 50
Angle BETA = 30
Magnitude of load P = 600
Increment of THETA = 10

THETA	TAB	TAC
-60.000	-0.000	600.000
-50.000	105.796	609.256
-40.000	208.378	600.000
-30.000	304.628	572.513
-20.000	391.622	527.631
(b) -10.000	466.717	466.717 (c)
0.000	527.631	391.622
10.000	572.513	304.628
20.000	600.000	208.378
30.000	609.256	105.796
40.000	600.000	-0.000

Set No.3
Angle ALPHA = 40
Angle BETA = 60
Magnitude of load P = 250
Increment of THETA = 5

THETA	TAB	TAC
-30.000	-0.000	250.000
-25.000	22.125	245.207
-20.000	44.082	238.547
-15.000	65.703	230.072
-10.000	86.824	219.846
-5.000	107.284	207.947
0.000	126.928	194.465
5.000	145.606	179.504
(b) 10.000	163.176	163.176 (c)
15.000	179.504	145.606
20.000	194.465	126.928
25.000	207.947	107.284
30.000	219.846	86.824
35.000	230.072	65.703
40.000	238.547	44.082
45.000	245.207	22.125
50.000	250.000	-0.000

2.C3



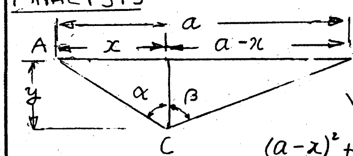
GIVEN:
LENGTH OF ROPE = $L = 20.1$ m
DISTANCE BETWEEN A AND B = $a = 20$ m
WEIGHT OF ACROBAT AND BALANCING POLE = $W = 800$ N
ASSUME NO SLIPPING AND NO ELASTIC DEFORMATION OF ROPE

FIND:
DEFLECTION y AND TENSIONS T_{AC} AND T_{BC} FOR VALUES OF x FROM 0.5 m TO 10.0 m, USING 0.5-m INCREMENTS
FROM DATA OBTAINED, ALSO FIND
(a) MAXIMUM DEFLECTION OF ROPE
(b) MAXIMUM TENSION IN ROPE
(c) SMALLEST VALUES OF T_{AC} AND T_{BC}

continued

2.C3 continued

ANALYSIS



FOR ANY x:

$$AC + CB = L$$

$$\sqrt{x^2+y^2} + \sqrt{(a-x)^2+y^2} = L$$

$$(a-x)^2 + y^2 = \left(L - \sqrt{x^2+y^2}\right)^2$$

$$a^2 - 2ax + x^2 + y^2 = L^2 - 2L\sqrt{x^2+y^2} + x^2 + y^2$$

$$2L\sqrt{x^2+y^2} = L^2 + 2ax - a^2$$

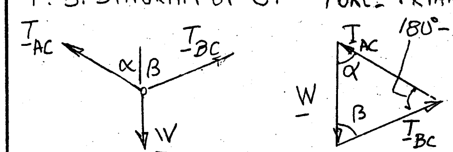
$$4L^2(x^2+y^2) = (L^2+2ax-a^2)^2$$

$$4L^2y^2 = (L^2+2ax-a^2)^2 - 4L^2x^2$$

$$y = \frac{1}{2L}\sqrt{(L^2+2ax-a^2)^2 - 4L^2x^2} \qquad (1)$$

ALSO: $\alpha = \tan^{-1}\left(\frac{x}{y}\right) \qquad \beta = \tan^{-1}\left(\frac{a-x}{y}\right) \qquad (2)$

F. B. DIAGRAM OF C: FORCE TRIANGLE:



LAW OF SINES:

$$\frac{T_{AC}}{\sin\beta} = \frac{T_{BC}}{\sin\alpha} = \frac{W}{\sin(\alpha+\beta)}$$

THUS: $T_{AC} = W\dfrac{\sin\beta}{\sin(\alpha+\beta)}, \qquad T_{BC} = W\dfrac{\sin\alpha}{\sin(\alpha+\beta)} \qquad (3)$

OUTLINE OF PROGRAM

ENTER $a = 20$ m, $L = 20.1$ m, $W = 800$ N

FOR $x = 0.5$ TO 10.1 USING 0.5 STEPS:
COMPUTE y FROM EQ. (1)
COMPUTE α AND β FROM EQS. (2)
COMPUTE T_{AC} AND T_{BC} FROM EQS. (3)

PROGRAM OUTPUT

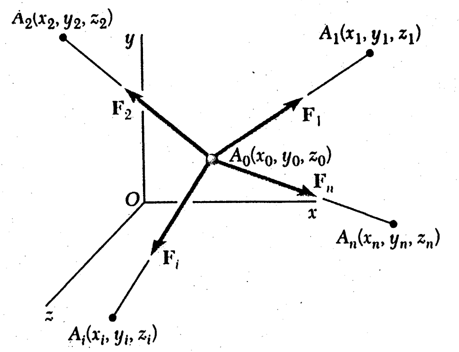
X	Y	TAC	TBC
0.5	0.327	1426.03	1193.97
1.0	0.446	1867.33	1706.14
1.5	0.534	2205.66	2078.69
2.0	0.606	2482.82	2377.48
2.5	0.666	2717.45	2627.65
3.0	0.718	2919.79	2842.03
3.5	0.764	3096.14	3028.22
4.0	0.803	3250.74	3191.13
4.5	0.838	3386.57	3334.19
5.0	0.869	3505.79	3459.85
5.5	0.895	3610.06	3569.95
6.0	0.919	3700.65	3665.90
6.5	0.939	3778.51	3748.76
7.0	0.956	3844.45	3819.40
7.5	0.970	3899.06	3878.49
8.0	0.981	3942.81	3926.55
8.5	0.990	3976.04	3963.95
9.0	0.996	3999.07	3991.06
9.5	1.000	4012.00	4008.01
10.0	1.001	4014.97	4014.97

(a) MAXIMUM DEFLECTION: $y_m = 1.001$ m ◄

(b) MAXIMUM TENSION:
FOR $x = 10.0$ m: $T_{AC} = T_{BC} = 4.01$ kN ◄

(c) SMALLEST VALUES OF T_{AC} AND T_{BC}:
FOR $x = 0.5$ m: $T_{AC} = 1.426$ kN ◄
$T_{BC} = 1.194$ kN ◄

2.C4



GIVEN: FORCES SHOWN, ACTING ON A_0.
WRITE PROGRAM TO DETERMINE THE MAGNITUDE AND DIRECTION OF THEIR RESULTANT.
APPLY PROGRAM TO SOLVE PROBS. 2.93, 2.94, 2.95, AND 2.135.

ANALYSIS

FIRST, FOR EACH FORCE $\underline{F}_i$ WE DETERMINE THE DISTANCE d_i FROM A_0 TO A_i:

$$d_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} \quad (1)$$

THE COMPONENTS OF $\underline{F}_i$ ARE

$$(F_x)_i = F_i \frac{x_i - x_0}{d_i}, \; (F_y)_i = F_i \frac{y_i - y_0}{d_i}, \; (F_z)_i = F_i \frac{z_i - z_0}{d_i} \quad (2)$$

THE COMPONENTS OF THE RESULTANT $\underline{R}$ ARE

$$R_x = \sum_{i=1}^{n} (F_x)_i, \; R_y = \sum_{i=1}^{n} (F_y)_i, \; R_z = \sum_{i=1}^{n} (F_z)_i \quad (3)$$

THE MAGNITUDE OF THE RESULTANT $\underline{R}$ IS

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} \quad (4)$$

AND ITS DIRECTION COSINES ARE

$$\lambda_x = R_x/R, \; \lambda_y = R_y/R, \; \lambda_z = R_z/R \quad (5)$$

THE ANGLES THAT $\underline{R}$ FORMS WITH THE AXES ARE

$$\theta_x = \cos^{-1}\lambda_x, \; \theta_y = \cos^{-1}\lambda_y, \; \theta_z = \cos^{-1}\lambda_z \quad (6)$$

WHERE VALUES BETWEEN 0 AND 180° SHOULD BE SELECTED

OUTLINE OF PROGRAM

ENTER PROBLEM NUMBER
ENTER COORDINATES x_0, y_0, z_0 OF POINT A_0
ENTER NUMBER OF FORCES n
FOR EACH FORCE $\underline{F}_i$:
- ENTER MAGNITUDE F_i
- ENTER COORDINATES x_i, y_i, z_i OF POINT A_i
- COMPUTE d_i FROM EQ. (1)
- COMPUTE $(F_x)_i, (F_y)_i, (F_z)_i$ FROM EQS. (2)

COMPUTE R_x, R_y, R_z FROM EQS. (3)
COMPUTE R FROM EQ. (4)
COMPUTE $\theta_x, \theta_y, \theta_z$ FROM EQS. (5) AND (6)
IF YOU OBTAIN A NEGATIVE VALUE FOR ANY OF THE ANGLES, ADD 180° TO THAT VALUE.

continued

2.C4 continued

PROGRAM OUTPUT

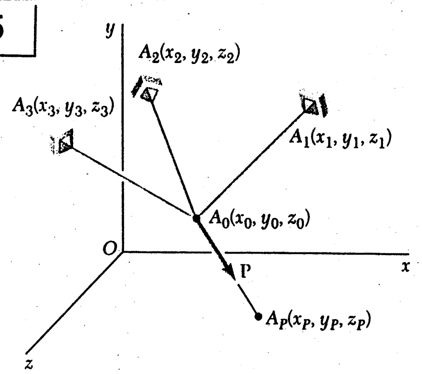
```
Data of Prob. 2.93
Coordinates of A0 ? -40,45,0
Number of given forces: N = 2
Magnitude of F(1)? 425
Coordinates of A(1)? 0,0,60
Magnitude of F(2)? 510
Coordinates of A(2)? 60,0,60
R = 912.92
THX =  48.2, THY = 116.6, THZ =  53.4
```

```
Data of Prob. 2.94
Coordinates of A0 ? -40,45,0
Number of given forces: N = 2
Magnitude of F(1)? 510
Coordinates of A(1)? 0,0,60
Magnitude of F(2)? 425
Coordinates of A(2)? 60,0,60
R = 912.92
THX =  50.6, THY = 117.6, THZ =  51.8
```

```
Data of Prob. 2.95
Coordinates of A0 ? 480,0,600
Number of given forces: N = 2
Magnitude of F(1)? 385
Coordinates of A(1)? 0,510,280
Magnitude of F(2)? 385
Coordinates of A(2)? 210,400,0
R = 747.83
THX = 120.1, THY =  52.5, THZ = 128.0
```

```
Data of Prob. 2.135
Coordinates of A0 ? 0,0,0
Number of given forces: N = 2
Magnitude of F(1)? 10
Coordinates of A(1)? -15.5885,15,12
Magnitude of F(2)? 7.5
Coordinates of A(2)? -15.5885,18.6,-15
R =  15.13
THX = 133.4, THY =  43.6, THZ =  86.6
```

2.C5



GIVEN:

THREE CABLES ATTACHED AT POINT A_0 AND FORCE P APPLIED AT A_0 AS SHOWN.

WRITE PROGRAM TO DETERMINE TENSION F_i IN EACH CABLE $(i = 1, 2, 3)$.

APPLY PROGRAM TO SOLVE PROBS. 2.102, 2.106, 2.107, 2.113 AND 2.115.

ANALYSIS

FIRST DETERMINE THE DISTANCE d FROM A_0 TO A_P:

$$d = \sqrt{(x_P - x_0)^2 + (y_P - y_0)^2 + (z_P - z_0)^2} \qquad (1)$$

THE COMPONENTS OF $\underline{P}$ ARE

$$P_x = \frac{P}{d}(x_P - x_0), \quad P_y = \frac{P}{d}(y_P - y_0), \quad P_z = \frac{P}{d}(z_P - z_0) \qquad (2)$$

NEXT DETERMINE FOR EACH CABLE A_0A_i $(i = 1, 2, 3)$ THE DIRECTION THE DISTANCE $d_i = A_0A_i$ AND THE DIRECTION COSINES $(\lambda_x)_i$, $(\lambda_y)_i$, $(\lambda_z)_i$:

$$d_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} \qquad (3)$$

$$(\lambda_x)_i = \frac{x_i - x_0}{d_i}, \quad (\lambda_y)_i = \frac{y_i - y_0}{d_i}, \quad (\lambda_z)_i = \frac{z_i - z_0}{d_i} \qquad (4)$$

THE COMPONENTS OF THE FORCE $\underline{F}_i$ EXERTED BY CABLE A_0A_i ON POINT A_0 ARE

$$(F_x)_i = F_i(\lambda_x)_i, \quad (F_y)_i = F_i(\lambda_y)_i, \quad (F_z)_i = F_i(\lambda_z)_i \qquad (5)$$

WE NOW WRITE THE EQUILIBRIUM EQUATIONS FOR A_0:

$$\sum_{i=1}^{3}(F_x)_i + P_x = 0, \quad \sum_{i=1}^{3}(F_y)_i + P_y = 0, \quad \sum_{i=1}^{3}(F_z)_i + P_z = 0$$

SUBSTITUTING FOR $(F_x)_i, (F_y)_i, (F_z)_i$ FROM (5) AND TRANSFERRING P_x, P_y, P_z:

$$(\lambda_x)_1 F_1 + (\lambda_x)_2 F_2 + (\lambda_x)_3 F_3 = -P_x$$
$$(\lambda_y)_1 F_1 + (\lambda_y)_2 F_2 + (\lambda_y)_3 F_3 = -P_y$$
$$(\lambda_z)_1 F_1 + (\lambda_z)_2 F_2 + (\lambda_z)_3 F_3 = -P_z$$

INTRODUCING THE DETERMINANT

$$\Delta = \begin{vmatrix} (\lambda_x)_1 & (\lambda_x)_2 & (\lambda_x)_3 \\ (\lambda_y)_1 & (\lambda_y)_2 & (\lambda_y)_3 \\ (\lambda_z)_1 & (\lambda_z)_2 & (\lambda_z)_3 \end{vmatrix} \qquad (6)$$

AND THE DETERMINANTS $\Delta_1, \Delta_2, \Delta_3$ OBTAINED BY SUBSTITUTING $-P_x, -P_y, -P_z$ SUCCESSIVELY FOR THE ELEMENTS OF THE FIRST, SECOND, AND THIRD COLUMN OF Δ, WE OBTAIN

$$F_1 = \frac{\Delta_1}{\Delta}, \quad F_2 = \frac{\Delta_2}{\Delta} \quad F_3 = \frac{\Delta_3}{\Delta} \qquad (7)$$

continued

2.C5 continued

OUTLINE OF PROGRAM

ENTER PROBLEM NUMBER
ENTER COORDINATES OF POINT A_0
ENTER MAGNITUDE OF LOAD P
ENTER COORDINATES OF A_P, A_1, A_2, AND A_3
USE EQS. (1) AND (2) TO COMPUTE COMPONENTS OF $\underline{P}$
USE EQS. (3) AND (4) TO COMPUTE λ_x, λ_y, λ_z FOR EACH CABLE
COMPUTE Δ FROM EQ. (6) AND $\Delta_1, \Delta_2, \Delta_3$
COMPUTE F_1, F_2, F_3 FROM EQS. (7) AND PRINT

REMARKS:

IN PROBS. 2.102, 2.113, AND 2.115, CHOOSE FOR A_P ANY POINT DIRECTLY ABOVE A.
IN PROB. 2.106, CHOOSE FOR A_P ANY POINT DIRECTLY UNDER A.
IN PROB. 2.107 CONSIDER P AS THE TENSION F_1 IN A FICTITIOUS CABLE PARALLEL TO THE x AXIS AND CHOOSE FOR A_1 ANY POINT DIRECTLY TO THE RIGHT OF A. ALSO CONSIDER THE TENSION IN AD AS THE GIVEN LOAD.

PROGRAM OUTPUT

```
Data of Prob. 2.102
Coordinates of point A0 ? 0,5.6,0
Magnitude of load: P = 800
Coordinates of point AP ? 0,10,0
Coordinates of point A1 ? -4.2,0,0
Coordinates of point A2 ? 2.4,0,4.2
Coordinates of point A3 ? 0,0,-3.3
F1 = 200.9  F2 = 371.7  F3 = 415.5

Data of Prob. 2.106
Coordinates of point A0 ? 0,-60,0
Magnitude of load: P = 1600
Coordinates of point AP ? 0,-100,0
Coordinates of point A1 ? -36,0,-27
Coordinates of point A2 ? 0,0,32
Coordinates of point A3 ? 40,0,-27
F1 = 570.9  F2 = 829.8  F3 = 527.5

Data of Prob. 2.107
Coordinates of point A0 ? 960,240,0
Magnitude of load: P = 305
Coordinates of point AP ? 0,960,-220
Coordinates of point A1 ? 1200,240,0
Coordinates of point A2 ? 0,0,380
Coordinates of point A3 ? 0,0,-320
F1 = 960.0  (F2 = 446.7)  (F3 = 341.7)

Data of Prob. 2.113
Coordinates of point A0 ? 0,100,0
Magnitude of load: P = 1800
Coordinates of point AP ? 0,200,0
Coordinates of point A1 ? -20,0,25
Coordinates of point A2 ? 60,0,18
Coordinates of point A3 ? -20,0,-74
F1 = 973.6  F2 = 531.0  F3 = 532.6

Data of Prob. 2.115
Coordinates of point A0 ? 0,480,0
Magnitude of load: P = 792
Coordinates of point AP ? 0,600,0
Coordinates of point A1 ? -320,0,360
Coordinates of point A2 ? 450,0,360
Coordinates of point A3 ? 250,0,-360
F1 = 510.0  F2 =  56.2  F3 = 536.3
```